SPIRAL AND WORM GEARING

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A TREATISE ON THE PRINCIPLES, DIMENSIONS, CALCULATION AND DESIGN OF SPIRAL AND WORM GEARING, TOGETHER WITH CHAPTERS ON THE METHODS OF CUTTING THE TEETH IN THESE TYPES OF GEARS

COMPILED AND EDITED

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PREFACE

THE manner in which MACHINERY'S book, "Spur and Bevel Gearing," has been received by the mechanical world has prompted the compilation and publication of a companion book on "Spiral and Worm Gearing." This subject has often been presented in so theoretical a manner that many have assumed it to be very difficult to master. It is possible, however, to present the principles of design and calculation of spiral and worm gearing in such a way that they can be readily understood without resorting to a highly theoretical treatment; and in preparing this book, the first consideration on the part of the editor has therefore been to treat the subject in such a way as to meet the practical requirements of the machine-building trade.

As a result, in this book, as well as in the companion book, "Spur and Bevel Gearing," mere theory and academic discussions have been avoided. The rules, formulas and instructions given are illustrated with engravings whenever necessary, and numerous examples are given to show their application to problems met with in machine design. Theoretical considerations, however, have not been neglected in cases where they have been found necessary to fully explain a practical process, and this book is, therefore, a treatise on both the theory and practice of spiral and worm gearing along such lines as will make it especially useful to practical men.

Readers of mechanical literature are familiar with MACHINERY'S 25-cent Reference Books which include the best of the material that has appeared in MACHINERY during the past years, adequately revised, amplified and brought up to date. Of these books, one hundred and thirty-five different titles have been published in the past seven years. Many subjects, however, cannot be adequately covered in all their phases in books of this size, and in answer to a demand for more comprehensive and detailed treatments of the more important mechanical subjects, it has been deemed advisable to bring out a number of larger volumes, each covering one subject completely. This book is one of these volumes.

The information contained in this book is mainly compiled from articles published in MACHINERY, and the best on the subject that has appeared in the Reference Books is also included, with necessary modifications and additions. For the material contained, MACHINERY is indebted to a large number of men who have furnished practical information to its columns. It has not been possible to give credit to each individual contributor in all instances, but it should be mentioned that the framework upon which the whole book has been built up consists of the Reference Books and articles which Mr. Ralph E. Flanders, the well-known gear expert and former associate editor of MACHINERY, has written and compiled; the chapter giving specific solutions for all the different cases of spiral gear problems has been contributed by Mr. J. H. Carver; to all other writers whose material has appeared in MACHINERY, and which is now used in this book, the publishers hereby express their appreciation.

MACHINERY.

NEW YORK, *September*, 1914.

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SPIRAL AND WORM GEARING

CHAPTER I

PRINCIPAL RULES AND FORMULAS FOR DESIGNING SPIRAL GEARS

THE subject of spiral or helical gearing is one which, from its very nature, can be approached by any one of a number of different ways, and it has been approached by so many of these possible different ways that the subject has, perhaps, become quite confused in the minds of many readers of technical literature.

The terms "spiral gear" and "helical gear" are, in usage, synonymous, but the former of these terms is, theoretically, incorrect. Inasmuch, however, as the word "spiral" is in such common use among mechanics in this connection, it has been used freely throughout this treatise.

Dimensions and Definitions. — Some of the terms used will require explanation. The center angle of a pair of helical or spiral gears is the angle made by the two center lines or axes of the gears, as taken in a view perpendicular to both axes. In Fig. 1 are shown views of three sets of spiral gears taken in the plane which shows the center angle. At the left is the ordinary case in which the shafts are at right angles with each other, so that the center angle (γ) is 90 degrees. In the second case γ is less than 90 degrees, and in the example shown at the right it is more. It should be noted in the last two cases that the position of the shaft axes is identical, but that the two center angles are located on opposite sides of axis A. In order to know on which side of the center line to take the center angle in cases like those shown, we have to reckon with the position of the teeth of the gears in contact. The center angle is taken at the side which includes the line x–x, passing lengthwise of the teeth of the gears at the point of contact with each other. Since the teeth are laid out differently in the two cases, the angles are different. The case shown in the center is the more usual of the two, the other being very rare.

In Fig. 2 is given a diagram showing what is meant by the "tooth angle" of a helical gear. In using the expression "tooth angle," the angle made by the tooth with the axis of the gear is meant, not the angle of the tooth with the face of the gear. Fig. 2 shows α_a as the tooth angle of gear a, and α_b as the tooth angle of gear b, used in the sense in which we will use them.

The number of teeth and the pitch diameter are terms which are identically the same as those used for spur gearing and,

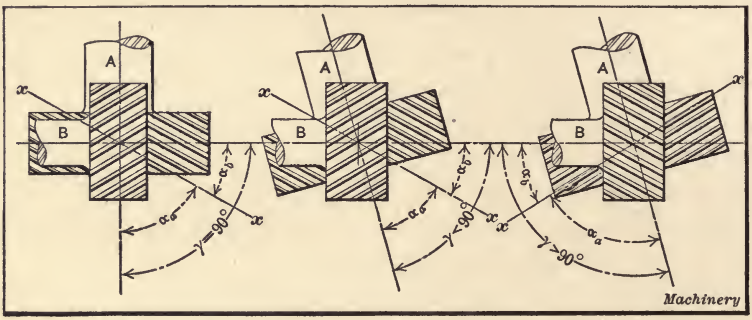


Fig. 1. Spiral Gears with Different Center Angles

therefore, require no explanation in this connection, it being a necessary assumption that anyone attempting to design a pair of spiral gears is well familiar with the design of spur gears. Practically all spiral gears are of small size, and hence are reckoned on the diametral pitch rather than the circular pitch system. All the rules and formulas given will, therefore, make use of the diametral pitch only. This may easily be found from the circular pitch by dividing 3.1416 by the circular pitch. The center distance is, of course, the shortest distance between the axes, and so is measured along the perpendicular common to both of them.

The regular diametral pitch of a spiral gear will be found, the same as for a spur gear, by dividing the number of teeth by the pitch diameter in inches. We are not interested in knowing what this is, however, since it does not enter into the calculations, and since the cutter used has to be for a somewhat finer diametral pitch. This is shown more clearly in Fig. 3. The normal diametral pitch, or diametral pitch of the cutter used, is reckoned from measurements taken along the pitch cylinder at right angles to the length of the tooth. P' represents the regular circular pitch, while P_n' represents the normal circular pitch. The diametral pitch may be found from this by dividing 3.1416 by P_n'. This is the pitch of the cutter to be used. The cutter, as will be explained in the following, cannot be selected for the

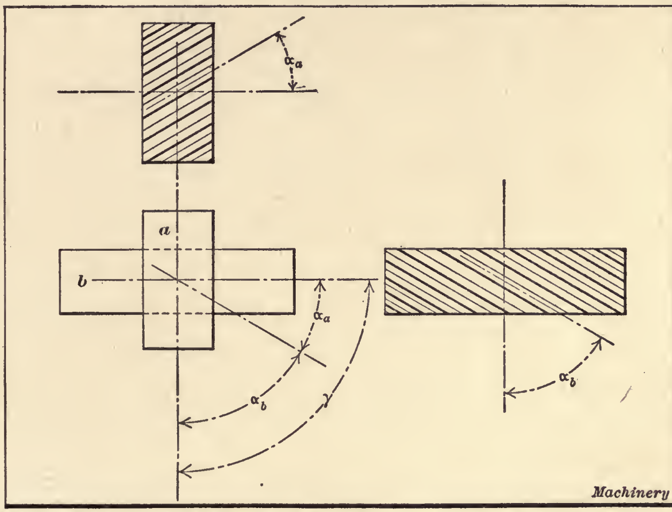


Fig. 2. Diagram showing Notation used for Tooth Angles

actual number of teeth in the gear, but must take into account the helix angle of the teeth as well, since the curvature as measured on a line at right angles to the helix is at a greater radius than when measured on the circle.

The length of the helix, or the lead, as shown in Fig. 3, is the length of pitch cylinder required to permit one complete revolution of the tooth if the latter were carried around for the full length of this cylinder. In Fig. 4 the relation of lead, circumference and tooth angle is plainly shown, the helix AB here being developed on a plane. The addendum S, and whole depth W of the tooth for helical gears, is the same as for plain spur gears. The normal thickness of tooth at the pitch line T_n, as shown in Fig. 3, is measured in a direction perpendicular to the face of the tooth. The regular tooth thickness is shown at T; this, however, does not enter into the calculations. The outside diameter, as for spur gears, is found by adding twice the addendum to the pitch diameter.

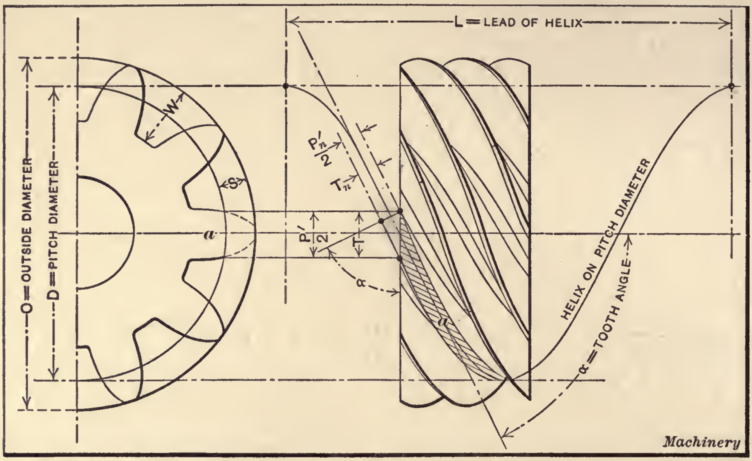


Fig. 3. Diagram of Spiral Gear Illustrating Terms used in the Calculations

Rules for Calculating Spiral Gear Dimensions. — The following rules are used for calculating the dimensions of spiral or helical gears:

Rule 1. The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle; that is to say, in Figs. 1 and 2 angle α_a added to α_b equals γ, as is self-evident from the engravings.

Rule 2. To find the pitch diameter of a helical gear, divide the number of teeth by the product of the normal pitch and the cosine of the tooth angle.

Rule 3. To find the center distance, add together the pitch diameters of the two gears and divide by 2. This rule is evidently the same as for spur gears.

Rule 4. To prove the calculations for pitch diameters and center distance, multiply the number of teeth in the first gear by the tangent of the tooth angle of that gear, and add the number of teeth in the second gear to the product; the sum should equal twice the product of the center distance multiplied by the normal diametral pitch, multiplied by the sine of the tooth angle of the first gear.

Rule 5. To find the number of teeth for which to select the cutter, divide the number of teeth in the gear by the cube of the cosine of the tooth angle.

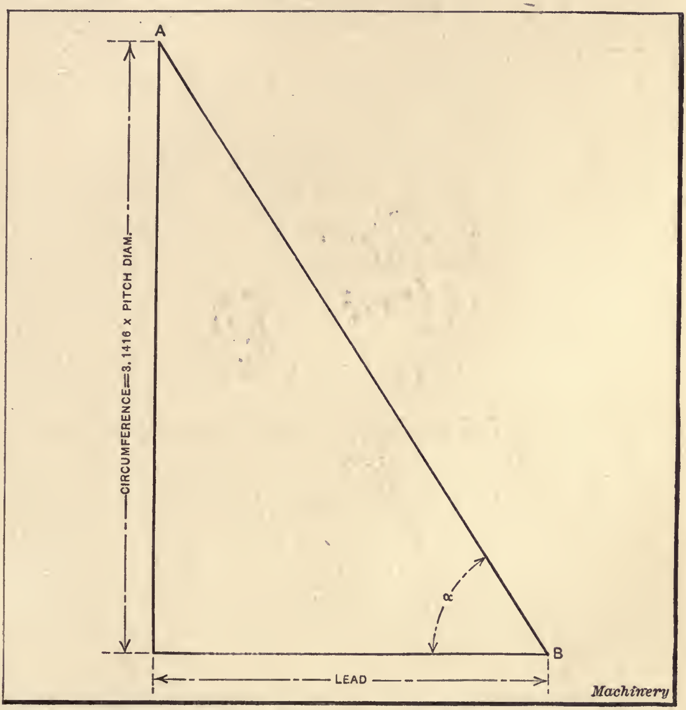


Fig. 4. Diagram showing Relation between Pitch Diameter, Lead and Helix Angle

Rule 6. To find the lead of the tooth helix, multiply the pitch diameter by 3.1416 times the cotangent of the tooth angle.

The rules relating to the addendum and the whole depth of tooth are the same as for spur gears. They are:

Rule 7. To find the addendum, divide 1 by the normal diametral pitch.

Rule 8. To find the whole depth of tooth space, divide 2.157 by the normal diametral pitch.

Rule 9. To find the normal tooth thickness at the pitch line, divide 1.571 by the normal diametral pitch.

Rule 10. To find the outside diameter, add twice the addendum to the pitch diameter.

The problem of designing a pair of spiral gears presents itself in general in two different forms or classes, which may be stated as follows:

Class 1. The diametral pitch and the numbers of teeth in the two gears are given.

Class 2. A fixed center distance is given, together with the velocity ratio or the numbers of teeth, with the requirement that standard cutters of even diametral pitch be used.

Examples of Calculations Under Class 1. — Let it be required to make the necessary calculations for a pair of spiral gears in which the shafts are at right angles. Normal diametral pitch equals 3; number of teeth in gear equals 45; number of teeth in pinion equals 18.

There being no restriction in this particular case as to center distance, we have to settle first on the tooth angles for the two gears. To obtain the highest efficiency, some authorities advise that the smallest tooth angle be given to the gear having the smallest number of teeth; and this angle should not, in general, run below 20 degrees. Keeping it nearly 30 or even up to 45 would be better. On the basis $\alpha_a = 30$ and $\alpha_b = 60$ degrees, we have the following calculations:

To find the pitch diameters, use Rule 2:

$$\text{Pitch diameter of gear} = \frac{45}{3 \times \cos 60^\circ} = 30 \text{ inches.}$$

$$\text{Pitch diameter of pinion} = \frac{18}{3 \times \cos 30^\circ} = 6.928 \text{ inches.}$$

To find the center distance, use Rule 3:

$$\frac{30 + 6.928}{2} = 18.464 \text{ inches.}$$

To prove that the previous calculations are correct, use Rule 4:

$$45 \times \tan 60^\circ + 18 = 95.940.$$
$$2 \times 18.464 \times 3 \times \sin 60^\circ = 95.939.$$

These two results are so nearly alike that the previous calculations may be considered fully correct.

To find the number of teeth for which to select the cutter, use Rule 5:

For gear, $\frac{45}{(\cos 60^\circ)^3} = 360.$

For pinion, $\frac{18}{(\cos 30^\circ)^3} = 28$, approximately.

To find the lead of the tooth helix, use Rule 6:

Lead for gear $= 3.1416 \times 30 \times \cot 60^\circ = 54.38$ inches.

Lead for pinion $= 3.1416 \times 6.928 \times \cot 30^\circ = 37.70$ inches.

To find the addendum, use Rule 7:

Addendum $= \frac{1}{3} = 0.333$ inch.

To find the whole depth of tooth space, use Rule 8:

Whole depth $= \frac{2.157}{3} = 0.719$ inch.

To find the normal tooth thickness at the pitch line, use Rule 9:

Tooth thickness $= \frac{1.571}{3} = 0.523$ inch.

To find the outside diameter, use Rule 10:

For gear, $30 + 0.666 = 30.666$ inches.

For pinion, $6.928 + 0.666 = 7.594$ inches.

This concludes the calculations for this example. If it is required that the pitch diameters of both gears be more nearly alike, the tooth angle of the gear must be decreased, and that of the pinion increased.

Suppose we have a case in which the requirements are the same as in Example 1, but it is required that both gears shall have the same tooth angle of 45 degrees. Under these conditions the addendum, whole depth of tooth and normal thickness at the pitch line would be the same, but the other dimensions would be altered as below:

Pitch diameter of gear $= \frac{45}{3 \times \cos 45^\circ} = 21.216$ inches.

Pitch diameter of pinion $= \frac{18}{3 \times \cos 45^\circ} = 8.487$ inches.

Center distance $= \frac{21.216 + 8.487}{2} = 14.851$ inches.

Number of teeth for which to select cutter:

For gear, $\frac{45}{(\cos 45^\circ)^3} = 127$, approximately.

For pinion, $\frac{18}{(\cos 45^\circ)^3} = 51$, approximately.

Lead of helix for gear $= 3.1416 \times 21.216 \times \cot 45^\circ$
$= 66.65$ inches.

Lead of helix for pinion $= 3.1416 \times 8.487 \times \cot 45^\circ$
$= 26.66$ inches.

Outside diameter of gear $= 21.216 + 0.666 = 21.882$ inches.

Outside diameter of pinion $= 8.487 + 0.666 = 9.153$ inches.

Examples of Calculations Under Class 2. — In Class 2 use is made of the term "equivalent diameter." The quotient obtained by dividing the number of teeth in a helical gear by the diametral pitch of the cutter used gives us a very useful factor for figuring the dimensions of helical gears, and this has been given the name "equivalent diameter," an abbreviation of the words "diameter of equivalent spur gear," which more accurately describe it. This quantity cannot be measured on the finished gear with a rule, being only an imaginary unit of measurement.

Rule 11. To find the equivalent diameter of a helical gear, divide the number of teeth of the gear by the diametral pitch of the cutter by which it is cut.

Preliminary Graphical Solution. — The process of locating a railway line over a mountain range is divided into two parts: the preliminary survey or period of exploration, and the final determination of the grade line. The problem of designing a pair of helical gears resembles this engineering problem in having many possible solutions, from which it is the business of the designer to select the most feasible. For the exploration or preliminary survey the diagram shown in Fig. 5 will be found a great convenience. The materials required are a ruler with a good straight edge, and a piece of accurately ruled, or, preferably, engraved, cross-section paper. If a point O be so located on the paper that BO, the distance to one margin line, be equal to the equivalent diameter of gear a, while $B'O$, the distance to the other margin line, be equal to the equivalent diameter of gear b, then (when the rule is laid diagonally across the paper in any position that cuts the margin lines and passes through point O) DO will be the pitch diameter of gear a, $D'O$ the pitch diameter of gear b, angle BOD the tooth angle of gear a and angle $B'OD'$ the tooth angle of gear b. This simple diagram presents

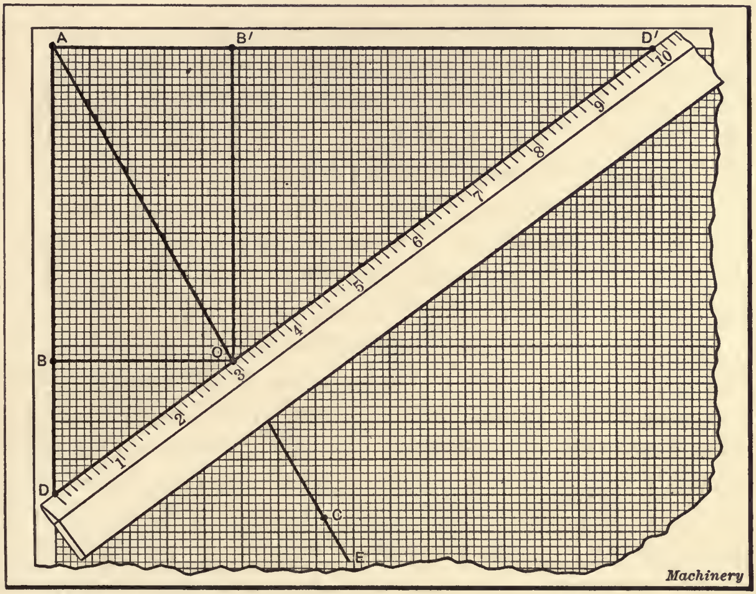


Fig. 5. Preliminary Solution with a Rule and Cross-section Paper

instantly to the eye all possible combinations for any given problem. It is, of course, understood that in the shape shown it can only be used for shafts making an angle of 90 degrees with each other.

The diagram as illustrated shows that a pair of helical gears having 12 and 21 teeth each, cut with a 5-pitch cutter, and having shafts at 90 degrees with each other and 5 inches apart, may have tooth angles of 36° 52′ and 53° 8′, and pitch diameters of 3 inches and 7 inches, respectively.

Suppose it were required to figure out the essential data for three sets of helical gears with shafts at right angles, as follows:

1st. Velocity ratio 2 to 1, center distance between shafts $2\frac{1}{4}$ inches.

2d. Velocity ratio 2 to 1, center distance between shafts $3\frac{3}{8}$ inches.

3d. Velocity ratio 2 to 1, center distance between shafts 4 inches.

We will take the first of these to illustrate the method of procedure about to be described.

We have a center distance of $2\frac{1}{4}$ inches and a speed ratio between driver and driven shafts of 2 to 1. The first thing to determine is the pitch of the cutter to use. The designer selects this according to his best judgment, taking into consideration the cutters on hand and the work the gearing will have to do. Suppose he decides that 12-pitch will be about right. In Fig. 5 it will be remembered that *DO* was the pitch diameter of gear *a*, while *D'O* was the pitch diameter of gear *b*. That being the case *DOD'* is equal to twice the distance between the shafts. In the problem under consideration this will be equal to $2 \times 2\frac{1}{4}$, or $4\frac{1}{2}$ inches. Fig. 6 is a skeleton outline showing the operation of making the preliminary survey with rule and cross-section paper. *AG* and *AG'* represent the margin lines of the sheet, while *DD'* represents the graduated straight-edge. By the conditions of the problem, the distance between points *D* and *D'*, where the ruler crosses the margin lines, must be equal to $4\frac{1}{2}$ inches. There has next to be determined at what angle of inclination the ruler shall be placed in locating this line. To do this we will first find our "ratio line." Select any point *C* such that *CF'* is to *CF* as 2 is to 1, which is the required ratio of the gears. Draw through point *C*, so located, the line *AE*. Line *AE* is then the ratio line, that is, a line so drawn that the measurements taken from any point on it to the margin lines will be to each other in the same ratio as the required ratio between the driving and driven gear. Now, by shifting the ruler on the margin lines, always being careful that they cut off the required distance of $4\frac{1}{2}$ inches on the graduations, it is found that when the rule is laid as shown in position No. 1, cutting the ratio line at O', the distance from the point of intersection to corner A is at its maximum. For the minimum value the tooth angle is the limiting feature. For a gear of this kind 30 degrees is, perhaps, about as small as would be advisable, so when the ruler is inclined at an angle of about 30 degrees with margin line AG', and occupies position No. 2 as shown, it will cut line AE at O'', and the distance cut off from the point of intersection

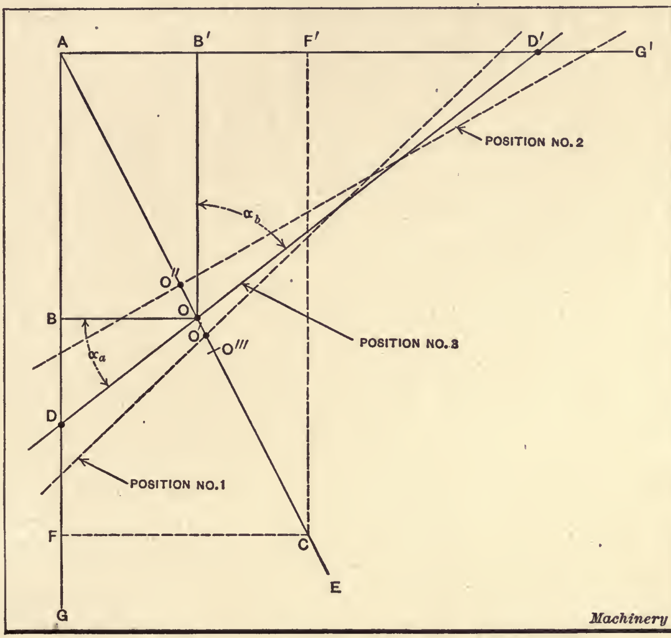


Fig. 6. Preliminary Graphical Solution for Problem No. 1

to corner A will be at its minimum value. The ruler must then be located at some intermediate position between No. 1 and No. 2.

Supposing, for example, 14 teeth in gear a and 28 teeth in gear b be tried. According to Rule 11 the equivalent diameter of gear a will then be $14 \div 12$, or 1.1666 inch; the equivalent diameter of b will be $28 \div 12$, or 2.3333 inches. Returning to the diagram to locate the point of intersection, it will be found that point O''' is so located that lines drawn from it to AG and

AG' will be equal to 1.1666 inch and 2.3333 inches, respectively, but this is beyond point O', which was found to be the outermost point possible to intersect with a $4\frac{1}{2}$-inch line DD'. This shows that the conditions are impossible of fulfillment.

Trying next 12 teeth and 24 teeth, respectively, for the two gears, the equivalent diameters by Rule 11 will be 1 inch and 2 inches. Point O is now so located that OB equals 1 inch and OB' equals 2 inches. Seeing that this falls as required between O' and O'', stick a pin in at this point to rest the straight-edge against, and shift the straight-edge about until it is located in such an angular position that the margin lines AG and AG' cut off $4\frac{1}{2}$ inches, or twice the required distance between the shafts, on the graduations. This gives the preliminary solution to the problem. Measuring as carefully as possible, DO, the pitch diameter of gear a, is found to be about 1.265 inch diameter, and $D'O$, the pitch diameter of gear b, about 3.235 inches. Angle BOD, the tooth angle of gear a, measures about 37° 50′. Angle $B'OD'$, the tooth angle of gear b, would then be 52° 10′ according to Rule 1.

Final Solution by Calculations. — To determine angle BOD more accurately than is feasible by a graphical process, use the following rule:

Rule 12. The tooth angle of gear a in a pair of mating helical gears a and b, whose axes are 90° apart, must be so selected that the equivalent diameter of gear b plus the product of the tangent of the tooth angle of gear a by the equivalent diameter of gear a will be equal to the product of twice the center distance by the sine of the tooth angle of gear a. (This rule, it will be seen, is simply a modification of Rule 4.)

That is to say in this case, $OB' + (OB \times$ the tangent of angle $BOD) = DD' \times$ the sine of angle BOD. Perform the operations indicated, using the dimensions which were derived from the diagram, to see whether the equality expressed in this equation holds true. Substituting the numerical values:

$$2 + (1 \times 0.77661) = 4.5 \times 0.61337,$$
$$2.77661 = 2.76016,$$

a result which is evidently inaccurate.

The solution of the problem now requires that other values for angle *BOD*, slightly greater or less than $37^\circ\ 50'$, be tried until one is found that will bring the desired equality. It will be found finally that if the value of $38^\circ\ 20'$ be used as the tooth angle of gear *a*, the angle is as nearly right as one could wish. Working out Rule 12 for this value:

$$2 + (1 \times 0.79070) = 4.5 \times 0.62024$$
$$2.79070 = 2.79108.$$

This gives a difference of only 0.00038 between the two sides of the equation. The final value of the tooth angle of gear *a* is thus settled as being equal to $38^\circ\ 20'$. Applying Rule 1 to find the tooth angle of gear *b* we have: $90^\circ - 38^\circ\ 20' = 51^\circ\ 40'$. The next rule relates to finding the pitch diameter of the gears.

Rule 13. The pitch diameter of a helical gear equals the equivalent diameter divided by the cosine of the tooth angle (or the equivalent diameter multiplied by the secant of the tooth angle). This rule is a modification of Rule 2.

If a table of secants is at hand, it will be somewhat easier to use the second method suggested by the rule, since multiplying is usually easier than dividing. Using in this case, however, the table of cosines, and performing the operation indicated by Rule 13, we have for the pitch diameter of gear *a*:

$$1 \div 0.78442 = 1.2748, \text{ or } 1.275 \text{ inch, nearly;}$$

and for the pitch diameter of gear *b*:

$$2 \div 0.62024 = 3.2245, \text{ or } 3.225 \text{ inches, nearly.}$$

To check up the calculations thus far, the pitch diameter of the two gears thus found may be added together. The sum should equal twice the center distance, thus:

$$1.275 + 3.225 = 4.500$$

which proves the calculations for the angle.

Applying Rule 10 to gear *a*:

$1.2748 + (2 \div 12) = 1.2748 + 0.1666 = 1.4414 = 1.441$ inch, nearly.

For gear *b*:

$3.2245 + (2 \div 12) = 3.2245 + 0.1666 = 3.3911 = 3.391$ inches, nearly.

In cutting spur gears of any given pitch, different shapes of cutters are used, depending upon the number of teeth in the gear to be cut. For instance, according to the Brown & Sharpe system for involute gears, eight different shapes are used for cutting the teeth in all gears, from a 12-tooth pinion to a rack. The fact that a certain cutter is suited for cutting a 12-tooth spur gear is no sign that it is suitable for cutting a 12-tooth helical gear, since the fact that the teeth are cut on an angle alters their shape considerably. To find out the number of teeth for which the cutter should be selected, use Rule 5.

Applying Rule 5 to gear *a*:

$$12 \div 0.784^3 = 12 \div 0.4818 = 25 -$$

and for gear *b*:

$$24 \div 0.620^3 = 24 \div 0.2383 = 100 +$$

giving, according to the Brown & Sharpe system, cutter No. 5 for gear *a* and cutter No. 2 for gear *b*.

In gearing up the head of the milling machine to cut these gears it is necessary to know the lead of the helix or "spiral" required to give the tooth the proper angle. To find this use Rule 6. In solving problems by this rule, as for Rule 5, it will be sufficient to use trigonometrical functions to three significant places only, this being accurate enough for all practical purposes. Solving by Rule 6 to find the lead for which to set up the gearing in cutting *a*:

$$1.275 \times 1.265 \times 3.14 = 5.065, \text{ or } 5\tfrac{1}{16} \text{ inches, nearly;}$$

for gear *b*:

$$3.225 \times 0.791 \times 3.14 = 8.010, \text{ or } 8\tfrac{3}{32} \text{ inches, nearly.}$$

The lead of the helix must be, in general, the adjustable quantity in any spiral gear calculation. If special cutters are to be made, the lead of the helix may be determined arbitrarily from those given in the milling machine table; this will, however, probably necessitate a cutter of fractional pitch. On the other hand, by using stock cutters and varying the center distance slightly, we might find a combination which would give us for one gear a lead found in the milling machine table, but it would only be chance that would make the lead for the helix in the mating gear also of standard length. It is then generally better to calculate the milling machine change gears according to the usual methods to suit odd leads, rather than to adapt the other conditions to suit an even lead. It will be found in practice that the lead of the helix may be varied somewhat from that calculated without seriously affecting the efficiency of the gears.

The remaining calculations relating to the proportions of the teeth do not vary from those for spur gears and are here set down for the sake of completeness only.

The addendum of a standard gear is found by Rule 7:

For gears *a* and *b* this will give:

$$1 \div 12 = 0.0833 \text{ inch.}$$

The whole depth of the tooth is found by Rule 8:

This gives for gears *a* and *b*:

$$2.157 \div 12 = 0.1797 \text{ inch.}$$

The thickness of the tooth is found by Rule 9:

For gears *a* and *b* of the problem this gives:

$$1.571 \div 12 = 0.1309 \text{ inch.}$$

Variations in Methods Used. — This completes all the calculations required to give the essential data for making our first pair of helical gears. To illustrate the variety of conditions for which these problems may be solved, the other cases will be worked out somewhat differently. In the case just considered no allowance was made for possible conditions which might have limited the dimensions of the gears, and the problem was solved for what might be considered good general practice. Gear *a*, however, might have been too small to put on the shaft on which it was intended to go, while gear *b* might have been too large to enter the space available for it. If, as we may assume, these gears are intended to drive the cam-shaft of a gas engine, the solution would probably be unsatisfactory. Case No. 2 will therefore be solved for a center distance of $3\frac{3}{8}$ inches as required, but the two gears will be made of about equal diameter.

Fig. 7 shows the preliminary graphical solution of this problem, the reference letters in all cases being the same as in Fig. 6. With a 10-pitch cutter, if this suited the judgment of the designer, 15 teeth in gear *a* and 30 teeth in gear *b* would require that the point of intersection on the ratio line *AE* be located at *O* where *BO* equals the equivalent diameter of gear *a*, which equals $1\frac{1}{2}$ inch, while *B'O* equals the equivalent diameter of gear *b*, or

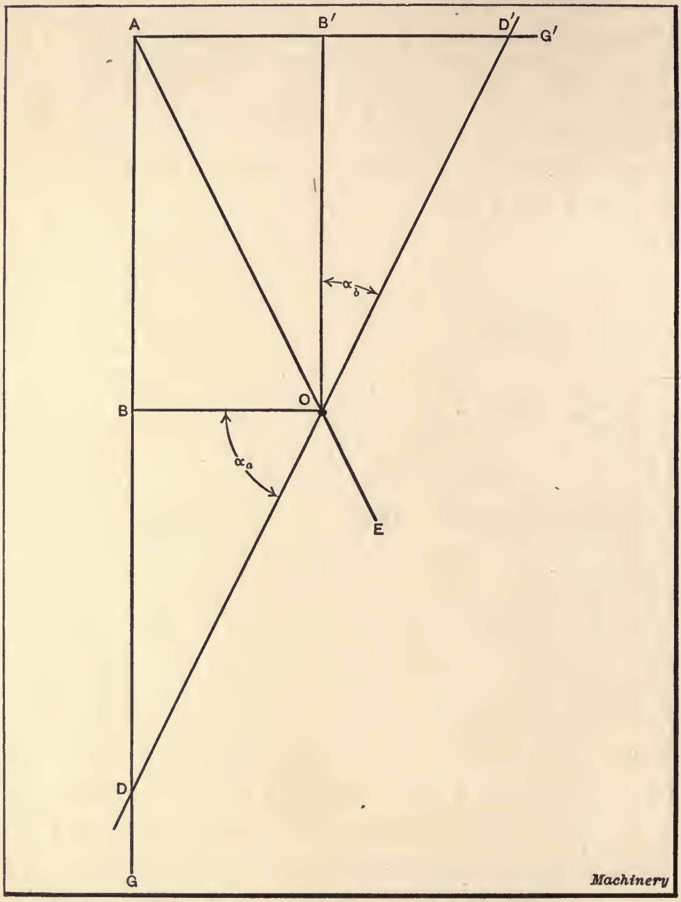


Fig. 7. Solution of Problem No. 2 for Equal Diameters

3 inches, both calculated in accordance with Rule 11. The required condition now is that *DO* be approximated to *D'O*; that is to say, that the pitch diameters of the two gears be about equal. After continued trial it will be found impossible to locate *O*, using a cutter of standard diametral pitch, so that *DO* and

$D'O$ shall be equal, and at the same time have DD' equal to twice the required center distance, which is $2 \times 3\frac{3}{8}$ inches or $6\frac{3}{4}$ inches. If this center distance could be varied slightly without harm, BD could be taken as equal to AB; then it would be found that a line drawn from D through O to D', though giving a somewhat shortened center distance, would make two gears of exactly the same pitch diameter.

Drawing line DOD', however, as first described to suit the conditions of the problem, and measuring it for a preliminary solution the following results are obtained: The tooth angle of gear a = angle $BOD = 63° 45'$; and the tooth angle of gear b = angle $B'OD' = 90° - 63° 45' = 26° 15'$, according to Rule 1. Performing the operations indicated in Rule 12 to correct these angles, it is found that when the tooth angle of gear a is $63° 54'$, and that for gear b is $26° 6'$, the equation of Rule 12 becomes:

$$3 + (15 \times 2.04125) = 6.75 \times 0.89803$$
$$6.06187 = 6.06170$$

which is near enough for all practical purposes. The other dimensions are easily obtained as before by using the remaining rules.

To still further illustrate the flexibility of the helical gear problem, the third case, for a center distance of 4 inches, will be solved in a third way. It is shown in MacCord's "Kinematics" that to give the least amount of sliding friction between the teeth of a pair of mating helical gears, the angles should be so proportioned that, in the diagrams, line DD' will be approximately at right angles to ratio line AE. On the other hand, to give the least end thrust against the bearings, line DD' should make an angle of 45 degrees with the margin lines AG and AG', in the case of gears with axes at an angle of 90 degrees, as are the ones being considered. The first example, worked out in detail, was solved in accordance with "good practice," and line DD' was located about one-half way between the two positions just described, thus giving in some measure the advantage of a comparative absence of sliding friction, combined with as small degree of end thrust as is practicable. To illustrate some of the peculiarities of the problem, Case 3 will now be solved to give the minimum amount of sliding friction, neglecting entirely the end thrust, which is considered to be taken up by ball thrust bearings or some equally efficient device.

By trial it will be found that, with the same number of teeth in the gear and with the same pitch as in Case 2, giving, in Fig. 8, *BO*, the equivalent diameter of gear *a*, a value of 1½ inch, and *B′O*, the equivalent diameter of gear *b*, a value of 3 inches, as in Fig. 7, line *DD′*, which is equal to twice the center distance, or 8 inches, can then lie at an angle of about 90 degrees with *AE*, thus meeting the condition required as to sliding friction. Thus this diagram, while relating to gears having the same pitch and number

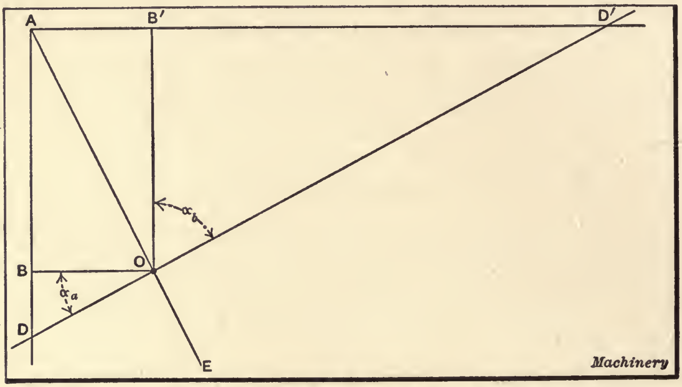


Fig. 8. Solution of Problem No. 3 for Minimum Sliding Friction

of teeth as Fig. 7, has an entirely different appearance, and gives different tooth angles and center distances, solving the problem as it does for the least sliding friction instead of for equal diameters of gears.

Measuring the diagram as accurately as may be, the following results are obtained: Tooth angle of gear a = BOD = 28°; tooth angle of gear b = angle $B'OD'$ = 90° − 28° = 62°. This is the preliminary solution. After accurately working it out by the process before described, we have as a final solution, tooth angle of gear a = 28° 28′; tooth angle of gear b = 61° 32′. From this the remaining data can be calculated.

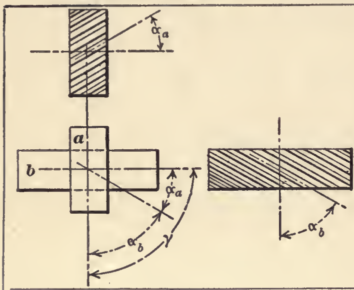
For designers who are skillful enough to solve such problems as these graphically without reference to calculations, the diagram may be used for the final solution. The variation between the results obtained graphically and those obtained in the more accurate mathematical solution is a measure of the skill of the draftsman as a graphical mathematician. The method is simple enough to be readily copied in a notebook or carried in the head. If the graphical method is to be used entirely, it will be best not to trust to the cross-section paper, which may not be accurately ruled; instead skeleton diagrams like those shown in Figs. 6, 7 and 8 may be drawn. For rough solutions, however, to be afterward mathematically corrected, as in the examples considered in this chapter, good cross-section paper is accurate enough. It permits of solving a problem without drawing a line. Point O may be located by reading the graduations; a pin inserted here may be used as a stop for the rule, from which the diameter and center distance are read directly; dividing AD, read from the paper, by DD', read from the rule, will give the sine of the tooth angle of the gear a.

Basic Rules and Formulas for Spiral Gears. — The rules and formulas given in the foregoing may be tabulated as shown on the following page. In the formulas in the table "Basic Rules and Formulas for Spiral Gear Calculations" the following notation is used:

P_n = normal diametral pitch (pitch of cutter);
D = pitch diameter;
N = number of teeth;
α = spiral angle;
γ = center angle, or angle between shafts;
C = center distance;
N' = number of teeth for which to select cutter;
L = lead of tooth helix;
S = addendum;
W = whole depth of tooth;
T_n = normal tooth thickness at pitch line;
O = outside diameter.

Examples of Spiral Gear Problems. — As proficiency in solving spiral gear problems can be obtained only by a great deal of practice, a number of examples will be given in the following,

Basic Rules and Formulas for Spiral Gear Calculations*



In the formulas N, α, etc., are the numbers of teeth, spiral angle, etc., for *either* gear or pinion; the notations N_a, N_b, α_a, α_b, etc., refer to the teeth or angles in the pinion or gear, respectively, in a pair of gears a and b.

No.	To Find	Rule	Formula
1	Relation between Shaft and Tooth Angles.	The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle.	$\gamma = \alpha_a + \alpha_b$
2	Pitch Diameter.	Divide the number of teeth by the product of the normal pitch and the cosine of the tooth angle.	$D = \frac{N}{P_n \cos \alpha}$
3	Center Distance.	Add together the pitch diameters of the two gears and divide by 2.	$C = \frac{D_a + D_b}{2}$
4	Checking Calculations in (2) and (3).	To prove the calculations for pitch diameters and center distance, multiply the number of teeth in the first gear by the tangent of the tooth angle of that gear, and add the number of teeth in the second gear to the product; the sum should equal twice the product of the center distance multiplied by the normal diametral pitch, multiplied by the sine of the tooth angle of the first gear.	$N_b + (N_a \times \tan \alpha_a) = 2\,CP_n \times \sin \alpha_a$
5	Number of Teeth for which to Select Cutter.	Divide the number of teeth in the gear by the cube of the cosine of the tooth angle.	$N' = \frac{N}{(\cos \alpha)^3}$
6	Lead of Tooth Helix.	Multiply the pitch diameter by 3.1416 times the cotangent of the tooth angle.	$L = \pi D \times \cot \alpha$
7	Addendum.	Divide 1 by the normal diametral pitch.	$S = \frac{1}{P_n}$
8	Whole Depth of Tooth.	Divide 2.157 by the normal diametral pitch.	$W = \frac{2.157}{P_n}$
9	Normal Tooth Thickness at Pitch Line.	Divide 1.571 by the normal diametral pitch.	$T_n = \frac{1.571}{P_n}$
10	Outside Diameter.	Add twice the addendum to the pitch diameter.	$O = D + 2S$

* From MACHINERY'S HANDBOOK.

which can be solved by simple modifications of the methods outlined for problems of Class 2. The same reference letters are used as before.

Example 1. — Find the essential dimensions for a pair of spiral gears, velocity ratio 3 to 1, center distance between shafts $5\frac{1}{8}$ inches, angle between shafts 38 degrees.

First obtain a preliminary solution by the diagram shown in Fig. 9. Draw lines AG and AG_1 making an angle γ with each other equal to 38 degrees, the angle between the axes. Locate the ratio line AE by finding any point such as O_1 between AG

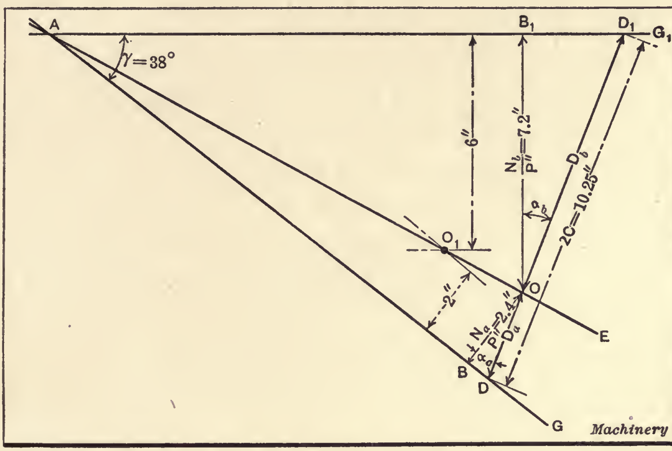


Fig. 9. Diagram Applying to the Solution of Example 1

and AG_1, that is distant from each of them in the same ratio as that desired for the gearing. In the case shown, it is 6 inches from AG_1 and 2 inches from AG, which is in the ratio of 3 to 1 as required. Through O_1 draw line AE which may be called the ratio line. Select a trial number of teeth and pitch of cutter for the two gears, such, for instance, as 36 teeth for the gear and 12 for the pinion, and with 5 diametral pitch of the cutter. The diameter of a spur gear of the same pitch and number of teeth as the gear would be $36 \div 5 = 7.2$ inches. Find the point O on AE, which is 7.2 inches from AG_1. This point will be 2.4 inches from AG, if AE is drawn correctly.

Now apply a scale to the diagram, with the edge passing through O and with the zero mark on line AG, shifting it to different positions until one is found in which the distance across from one line to another (DD_1 in the figure) is equal to twice the center distance, or 10.25 inches. If a position of the rule cannot be found which will give this distance between lines AG and AG_1, new assumptions as to number of teeth and diametral pitch of the gear and pinion must be made which will bring point O in a location where line DD_1 may be properly laid out. DD_1 being drawn, the problem is solved graphically. The tooth angle of the gear is B_1OD_1, or α_b, while that of the pinion is BOD, or α_a. OD_1 will be the pitch diameter of the gear, and OD the pitch diameter of the pinion.

To obtain the dimensions more accurately than can be done by the graphical process, the pitch diameters should be figured from the tooth angles we have just found. To do this, divide the dimensions OB_1 and OB for gear and pinion, by the cosine of the tooth angles found for them. If they measure on the diagram, for instance, 21 degrees 50 minutes and 16 degrees 10 minutes respectively (note that the sum of α_a and α_b must equal γ), the calculation will be as follows:

$$\begin{aligned} 7.2 \div 0.92827 &= 7.7563 = D_b \\ 2.4 \div 0.96046 &= \underline{2.4988 = D_a} \\ & 10.2551 = 2\,C \end{aligned}$$

The value we thus get, 10.2551 inches, for twice the center distance, is somewhat larger than the required value, 10.250 inches. We have now to assume other values for α_a and α_b, until we find those which give pitch diameters whose sum equals twice the center distance. Assume, for instance, that α_b = 21 degrees 43 minutes, then α_a = 38 degrees − 21 degrees 43 minutes = 16 degrees 17 minutes. We now have:

$$\begin{aligned} 7.2 \div 0.92902 &= 7.7501 = D_b \\ 2.4 \div 0.95989 &= \underline{2.5003 = D_a} \\ & 10.2504 = 2\,C \end{aligned}$$

This value for twice the center distance is so near that required that we may consider the problem as solved. The other dimensions for the outside diameter, lead, etc., may be obtained as for spiral gears at right angles, and as described in the previous part of this chapter.

Example 2. — Find the essential dimensions of a pair of spiral gears, velocity ratio 8 to 3, center distance between shafts $9\frac{5}{16}$ inches, angle between shafts 40 degrees.

The diagram for solving this problem is shown in Fig. 10. The axis lines AG_1 and AG are drawn as before and the ratio line AE is drawn in the ratio of 8 to 3, or 16 to 6, by the same method as just described. A point O is found having a location corresponding to 64 teeth and 5 pitch for the gear, and 24 teeth for the pinion. This gives distance OB_1 = 12.8 inches, and OB = 4.8 inches, by which position O is so located that a line DD_1 can be drawn through it at a convenient angle, and with a length equal to twice the center distance, or 18.625 inches. We measure the angle for a preliminary graphical solution as before, and then by trial find the final solution, in which angle α_b is 17 degrees 45 minutes, and α_a is 22 degrees 15 minutes as follows:

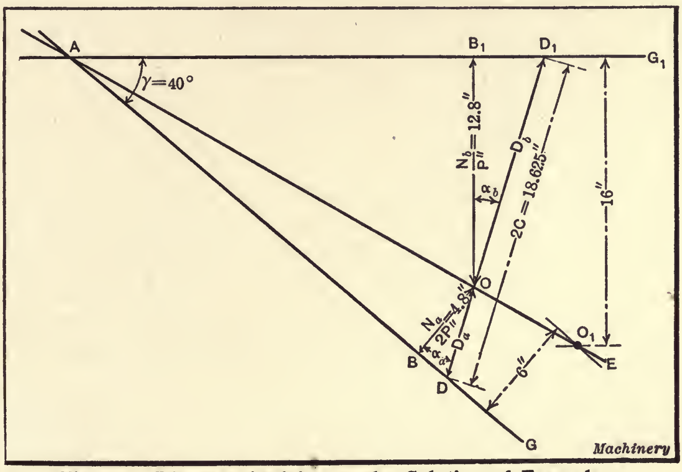


Fig. 10. Diagram Applying to the Solution of Example 2

$$12.8 \div 0.95240 = 13.4397 = D_b$$
$$4.8 \div 0.92554 = 5.1862 = D_a$$
$$18.6259 = 2\,C$$

This gives the value of twice the center distance near enough for gears of this size.

Example 3. — Find the essential dimensions for a pair of spiral gears, velocity ratio 5 to 2, center distance between shafts $4\frac{1}{16}$ inches, angle of shafts 18 degrees.

The diagram for solving this problem is shown in Fig. 11. The axis lines AG_1 and AG are drawn as before, and the ratio line AE is drawn in the ratio of 5 to 2, by the same method as just described. A point O is found having a location corresponding to 45 teeth and 8 pitch for the gear, and 18 teeth for the pinion. This gives distance $OB_1 = 5.625$ inches, and $OB = 2.250$ inches, in which position O is so located that line DD_1 can be drawn through it at a convenient angle, and with a length equal to twice the center distance, or 8.125 inches. We measure the angles for a preliminary mathematical solution as before, and

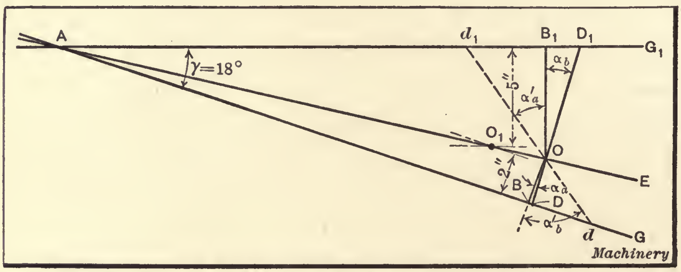
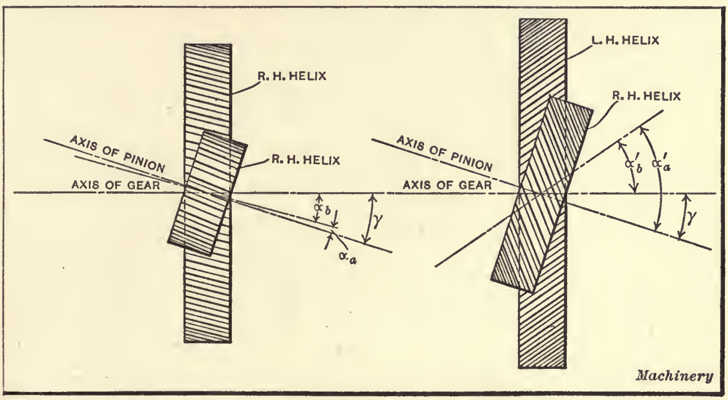


Fig. 11. Diagram Applying to the Solution of Example 3

then by trial find the final solution, in which angle α_b is 16 degrees 45 minutes and α_a is 1 degree 15 minutes as follows:

$$5.625 \div 0.95757 = 5.8742 = D_b$$
$$2.250 \div 0.99976 = 2.2505 = D_a$$
$$8.1247 = 2\,C$$

It is often a matter of great difficulty, when the center angle γ is as small as in this case, to find a location for point O such that standard cutters can be used, and that line DD_1 can be drawn of the proper length through O without bringing D to the left of B, or D_1 to the left of B_1. It will be noticed in this case that to make the center distance come right, angle α_a had to be made very small, so that the pinion is practically a spur gear. In some cases, to get the proper center distance, it may be necessary to so draw line DD_1 that one of the tooth angles is measured on the left side of BO or B_1O. Such a case, for instance, is shown in the position of d_1Od. When a line has to be drawn like this, the tooth angles α_a' and α_b' are opposite in inclination, instead of having them, as usual, either both right-hand or both left-hand. In Fig. 12 are shown gears drawn in accordance with the location of line DD_1 of Fig. 11, while Fig. 13 shows a pair drawn in accordance with dd_1 of the same diagram, which will illustrate the state of affairs met with in cases of this kind. This expedient of making one spiral gear right-hand and one left-hand should never be resorted to except in case of extreme necessity, as the construction involves a very wasteful amount of friction from the sliding of the teeth on each other as the gears revolve.



Figs. 12 and 13. Comparison between Two Pairs of Gears determined from the Diagram in Fig. 11

Demonstration of Grant's Formula. — As already mentioned, the number of teeth for which the cutter should be selected for cutting a helical gear, is found from the formula

$$N' = \frac{N}{\cos^3 \alpha}$$

in which N' = number of teeth for which cutter is selected;
N = actual number of teeth in helical gear;
α = angle of tooth with axis.

Note that $\cos^3 \alpha$ is equivalent to $(\cos \alpha)^3$.

A demonstration of this formula was presented by Mr. H. W.

Henes in MACHINERY, April, 1908. This demonstration is as follows:

Let P_n be the perpendicular distance between two consecutive teeth on the spiral gear, and let D_1 be the diameter of the spiral gear. Let the gear be represented as in Fig. 14, and pass a plane through it perpendicular to the direction of the teeth. The section will be an ellipse as shown in *CEDF*. Designate the semi-major and semi-minor axes by a and b, respectively.

Now N' is the number of teeth which a spur gear would have

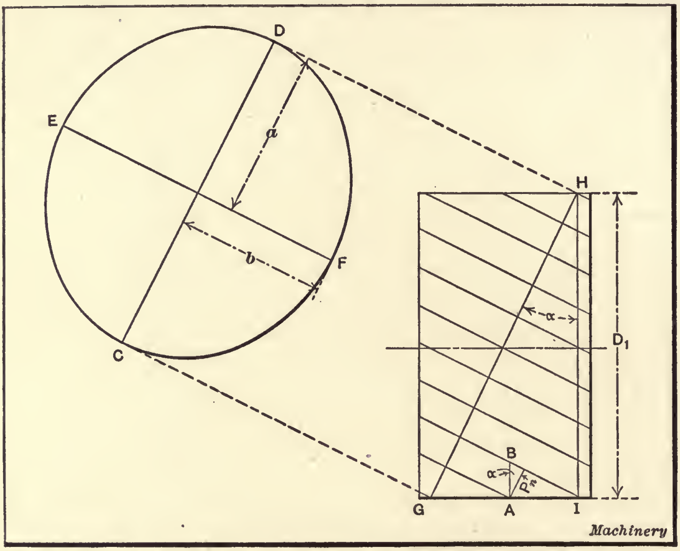


Fig. 14. Diagram for Deriving Formula for Determining Spur Gear Cutter to be used for Cutting Spiral Gears

if its radius were equal to the radius of curvature of the ellipse at *E*. Therefore, it is required to determine the radius of this curvature of the ellipse. This is done as follows:

From the figure we have:

$$2\,b = \text{axis } EF = D_1 \tag{1}$$

$$2\,a = \text{axis } CD = GH = \frac{HI}{\cos\alpha} = \frac{D_1}{\cos\alpha} \tag{2}$$

From (1) and (2) we have for a and b,

$$b = \frac{D_1}{2} \tag{3}$$

$$a = \frac{D_1}{2 \cos \alpha} \tag{4}$$

It is known, and shown by the methods of calculus, that the minimum curvature of an ellipse, that is, the curvature at E or F, equals $\frac{b}{a^2}$. Taking the values of a and b found in (3) and (4), we have the curvature at E:

$$\text{Curvature} = \frac{b}{a^2} = \frac{\frac{D_1}{2}}{\frac{D_1^2}{4 \cos^2 \alpha}} = \frac{4\, D_1 \cos^2 \alpha}{2\, D_1^2} = \frac{2 \cos^2 \alpha}{D_1} \tag{5}$$

It is also shown in calculus that the *curvature* is equal to $\frac{1}{R}$ where R is the *radius of curvature* at the point E. Therefore from (5) we have:

$$\frac{1}{R} = \frac{2 \cos^2 \alpha}{D_1} \quad \text{and thus} \quad R = \frac{D_1}{2 \cos^2 \alpha} \tag{6}$$

Formula (6) can also be arrived at directly, without reference to the minimum curvature of the ellipse, by introducing the formula for the radius of curvature in the first place. The curvature is simply the reciprocal value of the radius of curvature, and is only a comparative means of measurement. The radius of curvature of an ellipse at the end of its short axis is $\frac{a^2}{b}$, from which Formula (6) may be derived directly by introducing the values of a and b from Equations (3) and (4).

Having now found the radius of curvature of the ellipse at E, we proceed to find the number of teeth which a spur gear of that radius would have. From Fig. 14 we have:

$$AB = \frac{P_n}{\cos \alpha} \tag{7}$$

Now, if AB be multiplied by the number of teeth of the spiral gear, we shall obtain a quantity equal to the circumference of the gear; that is:

$$AB \times N = \pi D_1, \quad \text{and since} \quad AB = \frac{P_n}{\cos \alpha} \text{ from (7)}$$

$$\frac{P_n}{\cos \alpha} \times N = \pi D_1 \tag{8}$$

Since N' is the number of teeth which a spur gear of radius R would have, then,

$$N' = \frac{2\pi R}{P_n} \tag{9}$$

In Equation (9) the numerator of the fraction is the circumference of the spur gear whose radius is R, and the denominator is the circular pitch corresponding to the cutter.

From Equation (6) we have:

$$R = \frac{D_1}{2\cos^2\alpha}$$

Substituting this value of R in (9), we have:

$$N' = \frac{2\pi D_1}{P_n \times 2\cos^2\alpha} \tag{10}$$

From Equation (8) we have:

$$D_1 = \frac{NP_n}{\pi\cos\alpha} \tag{11}$$

Substitute this value of D_1 in Equation (10) and we have:

$$N' = \frac{2\pi NP_n}{2P_n\pi\cos^3\alpha}$$

or

$$N' = \frac{N}{\cos^3\alpha} \tag{12}$$

Since N is the number of teeth in a spiral gear and N' is the number of teeth in a spur gear which has the same radius as the radius of curvature of the ellipse referred to, this is the equivalent of saying that the cutter to be used should be correct for a number of teeth which can be obtained by dividing the actual number of teeth in the gear by the cube of the cosine of the tooth angle. Since the cosine of angle is always less than unity, its cube will be still less, so N' is certain to be greater than N, which will account for the fact that spiral gears of less than 12 teeth can be cut with the standard cutters.

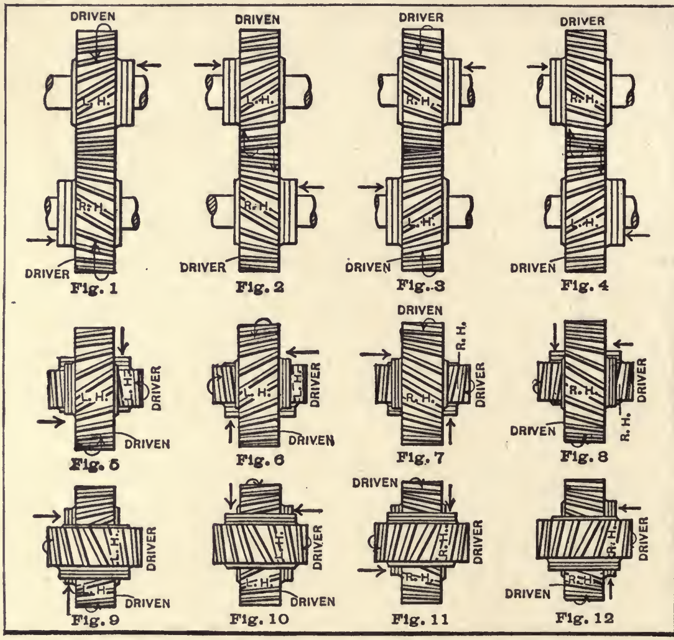
CHAPTER II

FORMULAS FOR SPECIAL CASES OF SPIRAL GEAR DESIGN

THE rules and formulas given in the tabulated arrangement in the preceding chapter are presented in the same order as they would ordinarily be used by the designer when calculating a pair of spiral gears. The formulas, however, cannot be directly applied to all cases of spiral gear problems, except by the use of a graphical method, as outlined, and a complete set of formulas for each of the sixteen different cases which are most frequently met with is, therefore, given in the following, together with an example for each case. These sixteen cases are:

1. Shafts parallel, ratio equal and center distance approximate.
2. Shafts parallel, ratio equal and center distance exact.
3. Shafts parallel, ratio unequal and center distance approximate.
4. Shafts parallel, ratio unequal and center distance exact.
5. Shafts at right angles, ratio equal and center distance approximate.
6. Shafts at right angles, ratio equal and center distance exact.
7. Shafts at right angles, ratio unequal and center distance approximate.
8. Shafts at right angles, ratio unequal and center distance exact.
9. Shafts at 45-degree angle, ratio equal and center distance approximate.
10. Shafts at 45-degree angle, ratio equal and center distance exact.
11. Shafts at 45-degree angle, ratio unequal and center distance approximate.

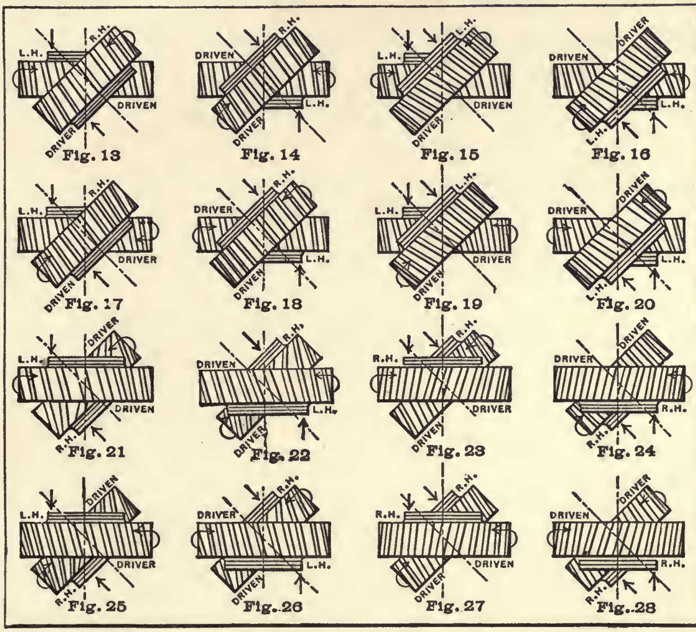
12. Shafts at 45-degree angle, ratio unequal and center distance exact.
13. Shafts at any angle, ratio equal and center distance approximate.
14. Shafts at any angle, ratio equal and center distance exact.
15. Shafts at any angle, ratio unequal and center distance approximate.
16. Shafts at any angle, ratio unequal and center distance exact.



Figs. 1 to 12. Thrust Diagrams for Spiral Gears — Direction of Thrust depends upon Direction of Rotation, Relative Position of Driver and Driven Gear, and Direction of Spiral

The proofs of the more complicated formulas are given in the explanatory matter preceding each specific set of formulas. All the information necessary for the shop operations is given, including the number of teeth marked on the spur gear cutter used, and the lead of the spiral, which data are often omitted on the drawing, but which always ought to be given. If omitted, the operator of the milling machine or gear cutter must determine these data himself, and this is not a commendable method.

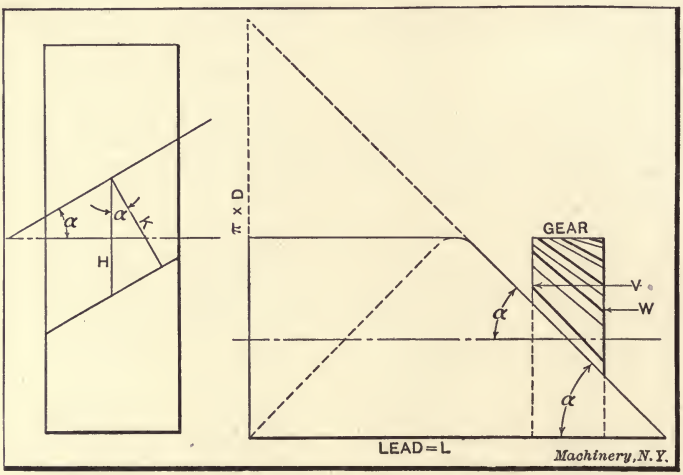
Procedure in Calculating Spiral Gears. — One of the first steps necessary in spiral gear design is to determine the direction of the thrust, if the thrust is to be taken in one direction only. When the direction of the thrust has been determined and the relative position of the driver and driven gear is known, the direction of spiral (right- or left-hand) may be found. The thrust diagrams, Figs. 1 to 28, are used for finding the direction of spiral. The arrows at the end bearings of the gears indicate



Figs. 13 to 28. Thrust Diagrams for Spiral Gears — Direction of Thrust depends upon Direction of Rotation, Relative Position of Driver and Driven Gear, and Direction of Spiral

the direction of the reaction against the thrust caused by the tooth pressure. The direction of the thrust depends on the direction of spiral, the relative positions of driver and driven gear and the direction of rotation. If the exact condition with regard to thrust is not found in the diagrams, it may be obtained by changing any one of these three conditions; that is, in Fig. 1 the thrust may be changed to the opposite direction by interchanging driver or driven gear, by reversing the direction of rotation or by changing the direction of spiral. Any one of these alterations will produce a thrust in the opposite direction.

The conditions of design will determine the nature of center distances, whether they must be exact or approximate. The strength of tooth needed, or sometimes the cutters on hand, will determine the normal pitch of the gear. The formulas given for the different conditions of spiral gearing are all based on the normal diametral pitch which is the same as the diametral pitch of the cutter used. The number of teeth in each gear is, of



Figs. 29 and 30. Diagrams for Derivation of Formulas

course, determined by the required speed ratio of the shafts. The angle of spiral depends on the conditions of the design, and the relative position of the shafts. If the shafts are parallel, the gears may be of the herringbone type, when an angle as great as 45 degrees may be used, as there is no end thrust. When used as shown in the thrust diagrams, the spiral angle should not exceed 20 degrees with parallel shafts, thus avoiding excessive end thrust. In order to obtain smooth running gears, the spiral angle should also be such that one end of the tooth remains in contact until the opposite end of the following tooth has found a bearing, as indicated at *V* and *W* in Fig. 30.

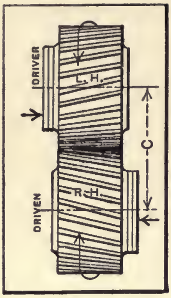
1. Shafts Parallel, Ratio Equal and Center Distance Approximate. — This case is met with in new designs, where an exact center distance is of no importance. The five factors, direction of spiral, approximate center distance, normal diametral pitch, number of teeth and angle of spiral are first determined upon. Then, from the formulas given, the required data are found as shown in the example given. The following shows the derivation of the formulas; in Fig. 29, let α be the angle of spiral with the axis of the gear; let H be the distance from one tooth to the next, measured on the circumference of the pitch circle, and K the normal circular pitch of the gear. Then $H = \frac{K}{\cos \alpha}$. The diametral pitch $= \frac{\pi}{\text{circular pitch}}$. Let P_n = normal diametral pitch, or diametral pitch of cutter used. Then $P_n = \frac{\pi}{K}$, or transposing, $K = \frac{\pi}{P_n}$. If N = number of teeth in gear, the circumference of the pitch circle $= N \times H$, and $\frac{NH}{\pi}$ = pitch diameter = D. Hence,

$$D = \frac{N}{\pi} \times \frac{K}{\cos \alpha} = \frac{N}{\pi \cos \alpha} \times \frac{\pi}{P_n} = \frac{N}{P_n \cos \alpha} \qquad (1)$$

In all cases where the shafts are parallel, the value of α is the same for both gears. The outside diameter of the spiral gear is found exactly as in spur gears, by adding $\frac{2}{P_n}$ to the pitch diameter. The derivation of formula $T = \frac{N}{\cos^3 \alpha}$ was treated fully in the preceding chapter.

A standard spur gear cutter is, of course, used in cutting spiral gears, but T, the number of teeth marked on it, will probably not be the actual number of teeth in the spiral gear to be cut, T depending, as the formula shows, on the spiral angle. As to the lead of spiral, let Fig. 30 represent a spiral gear, where the oblique line is the path of the tooth unfolded. Then $L = \pi D \cot \alpha$, in which L = lead of spiral, πD = pitch circumference and α = spiral angle.

Formulas, Case 1



Given or assumed:

1. Hand of spiral on driver or driven gear depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth.
5. α = angle of spiral (usually less than 20 degrees to avoid excessive end thrust).

To find:

1. D = pitch diameter $= \dfrac{N}{P_n \cos\alpha}$
2. O = outside diameter $= D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter $= \dfrac{N}{\cos^3\alpha}$
4. L = lead of spiral $= \pi D \cot\alpha$.

Example

Given or assumed:

1. See illustration.
2. $C_a = 3$ inches.
3. $P_n = 8$.
4. $N = 24$.
5. $\alpha = 15$ degrees.

To find:

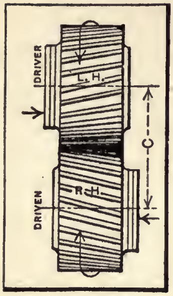
1. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{24}{8 \times 0.9659} = 3.106$ inches.
2. $O = 3.106 + \dfrac{2}{8} = 3.356$ inches.
3. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{24}{0.9} = 26.6$, say 27 teeth.
4. $L = \pi D \cot 15° = 3.1416 \times 3.106 \times 3.732 = 36.416$ inches.

2. Shafts Parallel, Ratio Equal and Center Distance Exact. — The spiral angle α is found in terms of the number of teeth in each gear, normal diametral pitch and pitch diameter, which latter is, of course, equal to the center distance.

From Formula (1) in Case (1), $D = \dfrac{N}{P_n \cos\alpha}$, or $\cos\alpha = \dfrac{N}{P_n \times D}$.

The remaining formulas are found as in the first case.

Formulas, Case 2



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending on rotation and direction in which thrust is to be received.
2. C = exact center distance = pitch diameter D.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth in each gear.

To find:

1. $\cos\alpha = \dfrac{N}{P_n D}$
2. O = outside diameter $= D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter $= \dfrac{N}{\cos^3\alpha}$
4. L = lead of spiral $= \pi D \cot\alpha$.

α is usually less than 20 degrees to avoid excessive end thrust.

Example

Given or assumed:

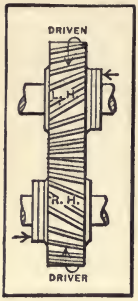
1. See illustration.
2. $C = 3$ inches.
3. $P_n = 8$.
4. $N = 22$.

To find:

1. $\cos\alpha = \dfrac{N}{P_n D} = \dfrac{22}{8 \times 3} = 0.9166$, or $\alpha = 23^\circ\ 34'$.
2. $O = D + \dfrac{2}{P_n} = 3 + \dfrac{2}{8} = 3\frac{1}{4}$ inches.
3. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{22}{(0.92)^3} = 28.2$, say 28 teeth.
4. $L = \pi D \cot\alpha = 3.1416 \times 3 \times 2.29 = 21.58$ inches.

3. Shafts Parallel, Ratio Unequal and Center Distance Approximate. — The formulas for this case are practically the same as for Case (1), and are derived in the same manner. The spiral angle is of the same value in both gears, as in all spiral gears with parallel shafts, but, of course, of a different direction (hand) in each gear.

Formulas, Case 3



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending upon rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch.
4. N = number of teeth in large gear.
5. n = number of teeth in small gear.
6. α = angle of spiral.

To find:

1. D = pitch diameter of large gear $= \dfrac{N}{P_n \cos \alpha}$
2. d = pitch diameter of small gear $= \dfrac{n}{P_n \cos \alpha}$
3. O = outside diameter of large gear $= D + \dfrac{2}{P_n}$
4. o = outside diameter of small gear $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter (large gear) $= \dfrac{N}{\cos^3 \alpha}$
6. t = number of teeth marked on cutter (small gear) $= \dfrac{n}{\cos^3 \alpha}$
7. L = lead of spiral on large gear $= \pi D \cot \alpha$.
8. l = lead of spiral on small gear $= \pi d \cot \alpha$.
9. C = center distance (if not right vary α) $= \frac{1}{2}(D + d)$.

Example

Given or assumed:

1. See illustration. 2. C_a = 17 inches. 3. P_n = 2.
4. N = 48. 5. n = 20. 6. α = 20 degrees.

To find:

1. $D = \dfrac{N}{P_n \cos \alpha} = \dfrac{48}{2 \times 0.9397} = 25.541$ inches.
2. $d = \dfrac{n}{P_n \cos \alpha} = \dfrac{20}{2 \times 0.9397} = 10.642$ inches.
3. $O = D + \dfrac{2}{P_n} = 25.541 + \dfrac{2}{2} = 26.541$ inches.

4. $o = d + \frac{2}{P_n} = 10.642 + \frac{2}{2} = 11.642$ inches.

5. $T = \frac{N}{\cos^3 \alpha} = \frac{48}{(0.9397)^3} = 57.8$, say 58 teeth.

6. $t = \frac{n}{\cos^3 \alpha} = \frac{20}{(0.9397)^3} = 24.1$, say 24 teeth.

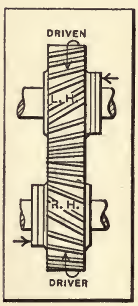
7. $L = \pi D \cot \alpha = 3.1416 \times 25.541 \times 2.747 = 220.42$ inches.

8. $l = \pi d \cot \alpha = 3.1416 \times 10.642 \times 2.747 = 91.84$ inches.

9. $C = \frac{1}{2}(D + d) = \frac{1}{2}(25.541 + 10.642) = 18.091$ inches.

4. Shafts Parallel, Ratio Unequal and Center Distance Exact. — In this case the sum of the two pitch diameters of the gears, or twice the center distance is $\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \cos \alpha} = 2\,C$ from which $\cos \alpha = \frac{N + n}{2\,P_n C}$.

N and n are the numbers of teeth in the respective gears, and C the center distance. The remaining eight formulas are similar to those of the other cases.



Formulas, Case 4

Given or assumed:

1. Position of gear having right- or left-hand spiral, depending upon rotation and direction in which thrust is to be received.
2. C = exact center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth in large gear.
5. n = number of teeth in small gear.

To find:

1. $\cos \alpha = \frac{N + n}{2\,P_n C}$

2. D = pitch diameter of large gear $= \frac{N}{P_n \cos \alpha}$

3. d = pitch diameter of small gear $= \frac{n}{P_n \cos \alpha}$

4. O = outside diameter of large gear $= D + \frac{2}{P_n}$

5. o = outside diameter of small gear $= d + \frac{2}{P_n}$

6. T = number of teeth marked on cutter (large gear) $= \frac{N}{\cos^3 \alpha}$

7. t = number of teeth marked on cutter (small gear) $= \frac{n}{\cos^3 \alpha}$

8. L = lead of spiral (large gear) $= \pi D \cot \alpha$.

9. l = lead of spiral (small gear) $= \pi d \cot \alpha$.

Example

Given or assumed:

1. See illustration. 2. $C = 18.75$ inches. 3. $P_n = 4$.
4. $N = 96$. 5. $n = 48$.

To find:

1. $\cos \alpha = \frac{N + n}{2 P_n C} = \frac{96 + 48}{2 \times 4 \times 18.75} = 0.96$, or $\alpha = 16° \ 16'$.

2. $D = \frac{N}{P_n \cos \alpha} = \frac{96}{4 \times 0.96} = 25$ inches.

3. $d = \frac{n}{P_n \cos \alpha} = \frac{48}{4 \times 0.96} = 12.5$ inches.

4. $O = D + \frac{2}{P_n} = 25 + \frac{2}{4} = 25.5$ inches.

5. $o = d + \frac{2}{P_n} = 12.5 + \frac{2}{4} = 13$ inches.

6. $T = \frac{N}{\cos^3 \alpha} = \frac{96}{(0.96)^3} = 108$ teeth.

7. $t = \frac{n}{\cos^3 \alpha} = \frac{48}{(0.96)^3} = 54$ teeth.

8. $L = \pi D \cot \alpha = 3.1416 \times 25 \times 3.427 = 269.15$ inches.

9. $l = \pi d \cot \alpha = 3.1416 \times 12.5 \times 3.427 = 134.57$ inches.

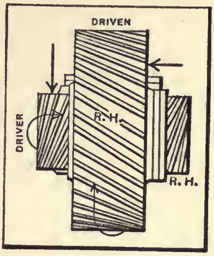
5. Shafts at Right Angles, Ratio Equal, and Center Distance Approximate. — The sum of the spiral angles of both gears must in this, and in the three following cases, equal 90 degrees, and the direction of the spiral must be the same for both gears. When the spiral angles of both gears equal 45 degrees, the pitch diameters of both gears will be equal, and are found by the formula:

$$D = \frac{N}{P_n \cos 45^\circ} = \frac{N}{0.70711\, P_n}$$

The other formulas are the same as those given in the previous cases.

Formulas, Case 5

When the spiral angles are 45 degrees, the gears are exactly alike; when other than 45 degrees, the sum of the spiral angles must equal 90 degrees.



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending on the rotation and direction in which the thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth.
5. α = angle of spiral.

To find:

(*a*) When spiral angles are 45 degrees.

1. D = pitch diameter $= \dfrac{N}{0.70711\, P_n}$
2. O = outside diameter $= D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter $= \dfrac{N}{0.353}$
4. L = lead of spiral $= \pi D$.
5. C = center distance $= D$.

(*b*) When spiral angles are other than 45 degrees.

1. D = pitch diameter $= \dfrac{N}{P_n \cos \alpha}$
2. T = number of teeth marked on cutter $= \dfrac{N}{\cos^3 \alpha}$
3. C = center distance = sum of pitch radii.
4. L = lead of spiral $= \pi D \cot \alpha$.

Example

Given or assumed:

1. See illustration. 2. $C_a = 2.5$ inches. 3. $P_n = 10$.
4. $N = 18$ teeth. 5. $\alpha = 45$ degrees.

To find:

1. $D = \dfrac{N}{0.70711\,P_n} = \dfrac{18}{0.70711 \times 10} = 2.546$ inches.

2. $O = D + \dfrac{2}{P_n} = 2.546 + \dfrac{2}{10} = 2.746$ inches.

3. $T = \dfrac{N}{\cos^3 \alpha} = \dfrac{18}{0.353} = 51$ teeth.

4. $L = \pi D \times 1 = 3.1416 \times 2.546 = 7.999$ inches.

6. Shafts at Right Angles, Ratio Equal and Center Distance Exact. — After deciding upon an approximate spiral angle of one gear, the number of teeth in each gear is found nearest the value $CP_n \cos \phi$, where ϕ is this approximate spiral angle, and C the exact center distance. If $\dfrac{N}{P_n \cos \alpha} = C$, or $N = CP_n \cos \alpha$, then N would be an approximate number of teeth with an exact spiral angle, but by making $\alpha = \phi$, an approximate angle, then $N = CP_n \cos \phi$, or an exact number of teeth, after which, as shown in the following, an exact angle α can be found, for

$$\frac{N}{P_n \cos \alpha} + \frac{N}{P_n \cos \beta} = 2\,C,$$

where α is the exact spiral angle of one gear, and β the exact spiral angle of the other, but $\beta = 90^\circ - \alpha$, or $\cos \beta = \sin \alpha$. Then,

$$\frac{N}{P_n \cos \alpha} + \frac{N}{P_n \sin \alpha} = 2\,C,$$

or

$$\frac{1}{\cos \alpha} + \frac{1}{\sin \alpha} = \frac{2\,CP_n}{N}$$

Multiplying by $\sin \alpha \cos \alpha$ gives:

$$\sin \alpha + \cos \alpha = \frac{2\,CP_n}{N} \sin \alpha \cos \alpha.$$

Squaring,

$$\sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha = \frac{4\,C^2 P_n^{\,2}}{N^2} \sin^2 \alpha \cos^2 \alpha.$$

But

$$2 \sin \alpha \cos \alpha = \sin 2\alpha, \text{ and } \sin^2 \alpha + \cos^2 \alpha = 1.$$

Further,

$$\sin^2 \alpha \cos^2 \alpha = \tfrac{1}{4} \sin^2 2\alpha.$$

Then

$$1 + \sin 2\alpha = \frac{C^2 P_n^2}{N^2} \sin^2 2\alpha,$$

or

$$\sin^2 2\alpha - \frac{N^2}{C^2 P_n^2} \sin 2\alpha = \frac{N^2}{C^2 P_n^2}$$

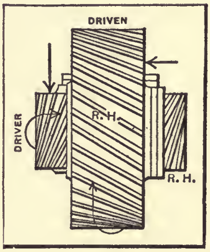
Solving this equation we get:

$$\sin 2\alpha = \frac{N^2}{2\,C^2 P_n^2} \pm \sqrt{\frac{N^2}{C^2 P_n^2} + \left(\frac{N^2}{2\,C^2 P_n^2}\right)^2}$$

which is the equation for finding twice the required angle.

Formulas, Case 6

Gears have same direction of spiral but probably different pitch diameters and spiral angles; the sum of the latter must be 90 degrees.



Given or assumed:

1. Position of gear having right- and left-hand spiral depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. ϕ = approximate spiral angle of one gear.
4. C = center distance.
5. N = number of teeth = nearest whole number to $CP_n \times \cos \phi$.

To find:

1. α = spiral angle of one gear.

$$\sin 2\alpha = \frac{N^2}{2\,C^2 P_n^2} \pm \sqrt{\frac{N^2}{C^2 P_n^2} + \left(\frac{N^2}{2\,C^2 P_n^2}\right)^2}$$

2. β = spiral angle of other gear = $90^\circ - \alpha$.
3. D = pitch diameter of one gear = $\dfrac{N}{P_n \cos \alpha}$

4. d = pitch diameter of other gear $= \dfrac{N}{P_n \cos\beta}$

5. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$

6. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$

7. T = number of teeth marked on cutter for one gear $= \dfrac{N}{\cos^3\alpha}$

8. t = number of teeth marked on cutter for other gear $= \dfrac{N}{\cos^3\beta}$

9. L = lead of spiral for one gear $= \pi D \cot\alpha$.

10. l = lead of spiral for other gear $= \pi d \cot\beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 10$. 3. $\phi = 45$ degrees.
4. $C = 4$ inches.
5. $N = CP_n \cos\phi = 4 \times 10 \times 0.70711 = 28.28$, say 28 teeth.

To find:

1. $\sin 2\alpha = 0.98664$, or $\alpha = 40° \; 19'$.
2. $\beta = 90° - \alpha = 49° \; 41'$.
3. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{28}{10 \times 0.76248} = 3.672$ inches.
4. $d = \dfrac{N}{P_n \cos\beta} = \dfrac{28}{10 \times 0.64701} = 4.328$ inches.
5. $O = 3.672 + 0.2 = 3.872$ inches.
6. $o = 4.328 + 0.2 = 4.528$ inches.
7. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{28}{(0.762)^3} = 63.6$, say 64 teeth.
8. $t = \dfrac{N}{\cos^3\beta} = \dfrac{28}{(0.647)^3} = 103.8$, say 104 teeth.
9. $L = \pi D \cot\alpha = 3.1416 \times 3.672 \times 1.1787 = 13.597$ inches.
10. $l = \pi d \cot\beta = 3.1416 \times 4.328 \times 0.84841 = 11.536$ inches.

7. Shafts at Right Angles, Ratio Unequal and Center Distance Approximate. — Here the only two terms which may not be decided upon at the outset are the number of teeth in each gear. Each, of course, must be a whole number and correspond with the center distance, angle of spiral and normal pitch. Let

$$\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \cos \beta} = 2\,C,$$

and

$$\frac{N}{n} = R, \quad \text{or} \quad N = Rn.$$

Then

$$\frac{Rn}{P_n \cos \alpha} + \frac{n}{P_n \cos \beta} = 2\,C.$$

Multiply by $P_n \cos \alpha \cos \beta$. Then

$$Rn \cos \beta + n \cos \alpha = 2\,CP_n \cos \alpha \cos \beta$$
$$n\,(R \cos \beta + \cos \alpha) = 2\,CP_n \cos \alpha \cos \beta,$$

and

$$n = \frac{2\,CP_n \cos \alpha \cos \beta}{R \cos \beta + \cos \alpha}$$

which gives the number of teeth in the pinion in terms of the center distance, angle of spiral and normal diametral pitch.

When a spiral angle of 45 degrees is used, the last formula becomes, by substituting the numerical values of the cosine of both angles, which is 0.70711:

$$n = \frac{2\,CP_n\, 0.70711 \times 0.70711}{R \times 0.70711 + 0.70711}$$

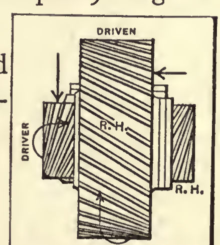
or

$$n = \frac{1.41\,CP_n}{R + 1}$$

Formulas, Case 7

Sum of spiral angles of gear and pinion must equal 90 degrees.

Given or assumed:



1. Position of gear having right- or left-hand spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. R = ratio of gear to pinion.

5. n = number of teeth in pinion = $\frac{1.41\, C_a P_n}{R + 1}$ for 45 degrees;

and $\frac{2\, C_a P_n \cos\alpha \cos\beta}{R \cos\beta + \cos\alpha}$ for any angle.

6. N = number of teeth in gear = nR.
7. α = angle of spiral on gear.
8. β = angle of spiral on pinion.

To find:

(*a*) When spiral angles are 45 degrees.

1. D = pitch diameter of gear = $\frac{N}{0.70711\, P_n}$
2. d = pitch diameter of pinion = $\frac{n}{0.70711\, P_n}$
3. O = outside diameter of gear = $D + \frac{2}{P_n}$
4. o = outside diameter of pinion = $d + \frac{2}{P_n}$
5. T = number of cutter (gear) = $\frac{N}{0.353}$
6. t = number of cutter (pinion) = $\frac{n}{0.353}$
7. L = lead of spiral on gear = πD.
8. l = lead of spiral on pinion = πd.
9. C = center distance (exact) = $\frac{D + d}{2}$

(*b*) When spiral angles are other than 45 degrees.

1. $D = \frac{N}{P_n \cos\alpha}$ 2. $d = \frac{n}{P_n \cos\beta}$
3. $T = \frac{N}{\cos^3\alpha}$ 4. $t = \frac{n}{\cos^3\beta}$
5. $L = \pi D \cot\alpha$ 6. $l = \pi d \cot\beta$

Example

Given or assumed:

1. See illustration. 2. C_a = 3.2 inches.
3. P_n = 10. 4. R = 1.5.
5. $n = \frac{1.41\, C_a P_n}{R + 1} = \frac{1.41 \times 3.2 \times 10}{1.5 + 1}$ = say 18 teeth.

6. $N = nR = 18 \times 1.5 = 27$ teeth.

7. $\alpha = 45$ degrees. 8. $\beta = 45$ degrees.

To find:

1. $D = \dfrac{N}{0.70711\,P_n} = \dfrac{27}{0.70711 \times 10} = 3.818$ inches.

2. $d = \dfrac{n}{0.70711\,P_n} = \dfrac{18}{0.70711 \times 10} = 2.545$ inches.

3. $O = D + \dfrac{2}{P_n} = 3.818 + \dfrac{2}{10} = 4.018$ inches.

4. $o = d + \dfrac{2}{P_n} = 2.545 + \dfrac{2}{10} = 2.745$ inches.

5. $T = \dfrac{N}{0.353} = \dfrac{27}{0.353} = 76.5$, say 76 teeth.

6. $t = \dfrac{n}{0.353} = \dfrac{18}{0.353} = 51$ teeth.

7. $L = \pi D = 3.1416 \times 3.818 = 12$ inches.

8. $l = \pi d = 3.1416 \times 2.545 = 8$ inches.

9. $C = \dfrac{D + d}{2} = \dfrac{3.818 + 2.545}{2} = 3.182$ inches.

8. Shafts at Right Angles, Ratio Unequal and Center Distance Exact. — This case is met with when two spiral gears are to replace two bevel gears, or when the conditions of the design demand an exact center distance and unequal ratio. The normal pitch, the ratio of number of teeth in large to small gear, the exact center distance and the approximate spiral angle α of the large gear are all given or assumed. Then the number of teeth in the small gear is found from the formula:

$$n = \frac{2\,CP_n \sin \alpha}{R \tan \alpha + 1}$$

which is found as follows:

Let

$$\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \cos \beta} = 2\,C,$$

or twice the center distance, or

$$\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \sin \alpha} = 2\,C.$$

Let $$\frac{N}{n} = R, \quad \text{or} \quad N = Rn.$$

Then $$\frac{Rn}{P_n \cos\alpha} + \frac{n}{P_n \sin\alpha} = 2C.$$

Multiply by $P_n \sin\alpha \cos\alpha$. Then

$$Rn \sin\alpha + n \cos\alpha = 2CP_n \sin\alpha \cos\alpha,$$
$$n(R \sin\alpha + \cos\alpha) = 2CP_n \sin\alpha \cos\alpha,$$

or $$n = \frac{2CP_n \sin\alpha \cos\alpha}{R \sin\alpha + \cos\alpha}$$

Divide by $\cos\alpha$. Then

$$n = \frac{2CP_n \sin\alpha}{R \tan\alpha + 1}$$

The formula $R \sec\alpha + \operatorname{cosec}\alpha = \frac{2CP_n}{n}$, which is used in finding the exact spiral angle, is found in the same manner as in some of the preceding cases. Let,

$$\frac{N}{P_n \cos\alpha} + \frac{n}{P_n \cos\beta} = 2C, \quad \text{and} \quad \frac{N}{n} = R, \quad \text{or} \quad N = Rn.$$

Then $$\frac{Rn}{P_n \cos\alpha} + \frac{n}{P_n \sin\alpha} = 2C.$$

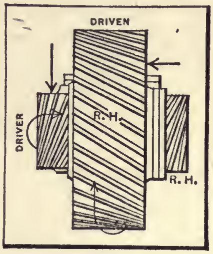
Multiplying by $\frac{P_n}{n}$ we have:

$$\frac{R}{\cos\alpha} + \frac{1}{\sin\alpha} = \frac{2CP_n}{n}$$

or $$R \sec\alpha + \operatorname{cosec}\alpha = \frac{2CP_n}{n}$$

This exact spiral angle is found by trial, by substituting values found in a table of secants and cosecants in the equation after the proper value of the last member in the equation has been found from known values. About 45-degree angles will probably be the most used, unless, for some reason of design, the spiral angle of one gear must be greater than that of the other. In using trigonometric tables to find values to satisfy the equation given, use first tenths only for trial, then hundredths, and so on, as the required value is approached. This shortens the work considerably. In solving this last equation, a table of functions, giving values to minutes, is necessary, as the value of its right-hand member must be to thousandths of an inch.

Formulas, Case 8



Gears have same direction of spiral. The sum of the spiral angles will equal 90 degrees.

Given or assumed:

1. Position of gear having right- or left-hand spiral depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. R = ratio of number of teeth in large gear to number of teeth in small gear.
4. α_a = approximate spiral angle of large gear.
5. C = exact center distance.

To find:

1. n = number of teeth in small gear nearest

$$\frac{2\,CP_n \sin\alpha_a}{1 + R\tan\alpha_a}$$

2. N = number of teeth in large gear = Rn.
3. α = exact spiral angle of large gear, found by trial from

$$R\sec\alpha + \operatorname{cosec}\alpha = \frac{2\,CP_n}{n}$$

4. β = exact spiral angle of small gear = $90° - \alpha$.
5. D = pitch diameter of large gear $= \dfrac{N}{P_n \cos\alpha}$
6. d = pitch diameter of small gear $= \dfrac{n}{P_n \cos\beta}$
7. O = outside diameter of large gear $= D + \dfrac{2}{P_n}$
8. o = outside diameter of small gear $= d + \dfrac{2}{P_n}$
9. T = number of teeth marked on cutter for large gear $= \dfrac{N}{\cos^3\alpha}$

10. t = number of teeth marked on cutter for small gear $= \dfrac{n}{\cos^3 \beta}$

11. L = lead of spiral on large gear $= \pi D \cot \alpha$.

12. l = lead of spiral on small gear $= \pi d \cot \beta$.

Example

Given or assumed:

1. See illustration 2. $P_n = 8$. 3. $R = 3$.
4. $\alpha_a = 45$ degrees. 5. $C = 10$ inches.

To find:

1. $n = \dfrac{2\,CP_n \sin \alpha_a}{1 + R \tan \alpha_a} = \dfrac{2 \times 10 \times 8 \times 0.70711}{1 + 3} = 28.25$, say 28 teeth.

2. $N = Rn = 3 \times 28 = 84$ teeth.

3. $R \sec \alpha + \operatorname{cosec} \alpha = \dfrac{2\,CP_n}{n} = \dfrac{2 \times 10 \times 8}{28} = 5.714$, or $\alpha = 46^\circ\ 6'$.

4. $\beta = 90^\circ - \alpha = 90^\circ - 46^\circ\ 6' = 43^\circ\ 54'$.

5. $D = \dfrac{N}{P_n \cos \alpha} = \dfrac{84}{8 \times 0.6934} = 15.143$ inches.

6. $d = \dfrac{n}{P_n \cos \beta} = \dfrac{28}{8 \times 0.72055} = 4.857$ inches.

7. $O = D + \dfrac{2}{P_n} = 15.143 + 0.25 = 15.393$ inches.

8. $o = d + \dfrac{2}{P_n} = 4.857 + 0.25 = 5.107$ inches.

9. $T = \dfrac{N}{\cos^3 \alpha} = \dfrac{84}{0.333} =$ say 252 teeth.

10. $t = \dfrac{n}{\cos^3 \beta} = \dfrac{28}{0.374} =$ say 75 teeth.

11. $L = \pi D \cot \alpha = 3.1416 \times 15.143 \times 0.96232 = 45.78$ inches.

12. $l = \pi d \cot \beta = 3.1416 \times 4.857 \times 1.0392 = 15.857$ inches.

Shafts at a 45-degree Angle. — In the following four cases formulas will be given for calculating spiral gears with a shaft angle of 45 degrees. As seen in Fig. 31 the treatment, as far as the design is concerned, will be the same for a 135-degree shaft angle as for an angle of 45 degrees. Thrust diagrams, Figs. 13 to 28, are given as an aid in determining the direction of thrust and rotation, and the direction of spiral, whether right- or left-hand. The arrows shown indicate the direction of the reaction against the thrust caused by the tooth pressure.

The relation between the direction of rotation, direction of spiral and spiral angle may be studied in Figs. 32 and 33. In Fig. 32 two spiral gears are shown, one in front of the other, with shafts at an angle of 45 degrees to each other. Line *AB* represents a right-hand spiral tooth on the front side of gear *C*. Assume gear *C* to rotate in the direction shown; then when the tooth *AB* reaches the rear side, it will be represented by line *EF*, which also represents the tooth direction on the front side of gear *G*. Angle *BOH* equals angle *EOH*, and it will be seen directly that the spiral angle of either gear equals 45 degrees minus the spiral angle of the other, both gears being right-hand. In Fig. 33 the spiral angles are shown to be of opposite hand, and one spiral angle is 45 degrees plus the other angle. From these illustrations we may draw the following conclusions relative to gears with a shaft angle of 45 degrees:

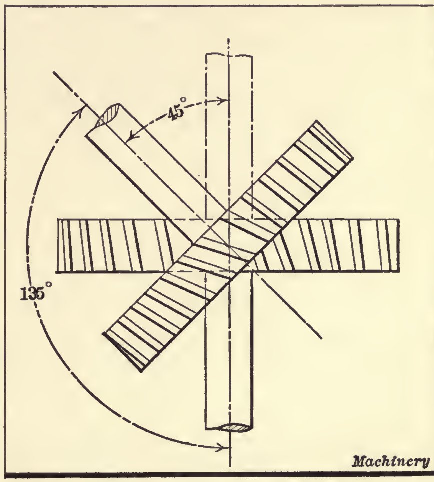


Fig. 31. Diagrammatical View showing General Arrangement of Spiral Gearing with Shafts at a 45-degree Angle

When the spiral angle of either gear is less than 45 degrees, then the spiral angles are the same hand, and one spiral angle is 45 degrees minus the other. When the spiral angle of either gear is greater than 45 degrees, then the spiral angles are of opposite hand, and the spiral angle of one gear is 45 degrees plus the spiral angle of the other.

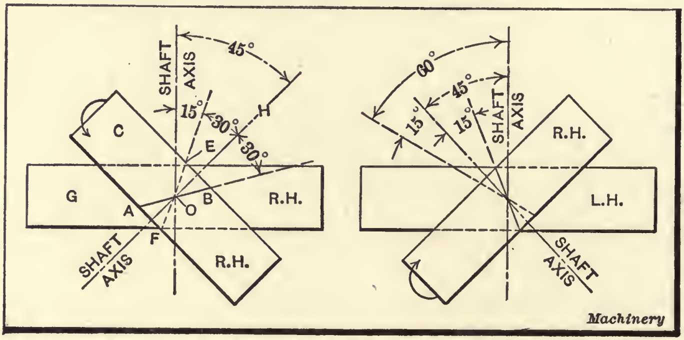
9. Shafts at a 45-degree Angle, Ratio Equal and Center Distance Approximate. — As already stated, the spiral angle of one gear must equal 45 degrees plus or minus the spiral angle of the other. The formulas in Part (*a*) in the following are to be used when the spiral angles of both gears are $22\frac{1}{2}$ degrees, which will often be the case. The pitch diameter of both gears will be equal, and are found by the formula (see below for notation):

$$D = \frac{N}{P_n \cos 22\frac{1}{2}^\circ} = \frac{N}{0.92388\, P_n}$$

Further

$$T = \frac{N}{\cos^3 22\frac{1}{2}^\circ} = \frac{N}{0.788}$$

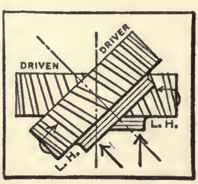
$$L = \pi D \cot 22\frac{1}{2}^\circ = 7.584\, D.$$



Figs. 32 and 33. Relation between the Spiral Angles of Teeth in Two Gears

The other formulas are the same as those given in the preceding cases. Part (*b*) is used for unequal spiral angles.

Formulas, Case 9



The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.

Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.

2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. α = angle of spiral of driving gear.
5. β = angle of spiral of driven gear.
6. N = number of teeth nearest $\dfrac{2\,C_a P_n \cos\alpha\cos\beta}{\cos\alpha + \cos\beta}$

To find:

(*a*) When spiral angles are $22\frac{1}{2}$ degrees.

1. D = pitch diameter $= \dfrac{N}{0.9239\,P_n}$
2. O = outside diameter $= D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter $= N \div 0.788$.
4. L = lead of spiral $= 7.584\,D$.
5. C = center distance $= D$.

(*b*) When spiral angles are other than $22\frac{1}{2}$ degrees.

1. D = pitch diameter of driver $= \dfrac{N}{P_n \cos\alpha}$
2. d = pitch diameter of driven gear $= \dfrac{N}{P_n \cos\beta}$
3. O = outside diameter of driver $= D + \dfrac{2}{P_n}$
4. o = outside diameter of driven gear $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter for driver $= N \div \cos^3\alpha$.
6. t = number of teeth marked on cutter for driven gear $= N \div \cos^3\beta$.
7. L = lead of spiral for driver $= \pi D \cot\alpha$.
8. l = lead of spiral for driven gear $= \pi d \cot\beta$.
9. C = actual center distance = sum of pitch radii.

Example

Given or assumed:

1. See illustration.
2. C_a = 4 inches.
3. P_n = 10.
4 and 5. $\alpha = \beta = 22\frac{1}{2}$ deg.
6. N = 37.

To find:

1. $D = \dfrac{N}{0.9239\,P_n} = \dfrac{37}{0.9239 \times 10} = 4.005$ inches.

2. $O = D + \dfrac{2}{P_n} = 4.005 + \dfrac{2}{10} = 4.205$ inches.

3. $T = N \div 0.788 = 37 \div 0.788 = 47$ teeth.

4. $L = 7.584\,D = 7.584 \times 4.005 = 30.374$ inches.

10. Shafts at a 45-degree Angle, Ratio Equal and Center Distance Exact. — Following the same method of reasoning as in Case (6), the approximate number of teeth in each gear is found from the equation:

$$\frac{N}{P_n \cos \alpha_a} + \frac{N}{P_n \cos \beta_a} = 2\,C,$$

from which, multiplying by $P_n \cos \alpha_a \cos \beta_a$,

$$N \cos \beta_a + N \cos \alpha_a = 2\,CP_n \cos \alpha_a \cos \beta_a, \text{ or}$$

$$N = \frac{2\,CP_n \cos \alpha_a \cos \beta_a}{\cos \beta_a + \cos \alpha_a}$$

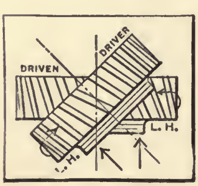
After the exact number of teeth to be used in each gear is found from the last equation, the spiral angles are found from the same equation used in finding the approximate number of teeth.

Let $\dfrac{N}{P_n \cos \alpha} + \dfrac{N}{P_n \cos \beta} = 2\,C$, where α and β are now the exact spiral angles. The secant being the reciprocal of the cosine,

$$\frac{N}{P_n} \sec \alpha + \frac{N}{P_n} \sec \beta = 2\,C, \text{ or } \sec \alpha + \sec \beta = \frac{2\,CP_n}{N}.$$

By using a table of secants, reading to minutes, angles can be found to satisfy this equation, after very few trials.

Formulas, Case 10



The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.

Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. C = center distance.
4. α_a = approximate spiral angle of one gear.
5. β_a = approximate spiral angle of the other gear.
6. N = number of teeth nearest $\dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a}$

To find:

1. α and β = exact spiral angles found by trial from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N}$
2. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos\alpha}$
3. d = pitch diameter of the other gear $= \dfrac{N}{P_n \cos\beta}$
4. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$
5. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$
6. T = number of teeth marked on cutter for one gear $= N \div \cos^3\alpha$.
7. t = number of teeth marked on cutter for other gear $= N \div \cos^3\beta$.
8. L = lead of spiral for one gear $= \pi D \cot\alpha$.
9. l = lead of spiral for other gear $= \pi d \cot\beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 8$. 3. C = 10 inches.
4. $\alpha_a = 15°$. 5. $\beta_a = 30°$.
6. $N = \dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a} = \dfrac{2 \times 10 \times 8 \times 0.96593 \times 0.86603}{0.96593 + 0.86603}$ = 73 teeth.

To find:

1. α and β from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N} = \dfrac{2 \times 10 \times 8}{73} = 2.1918$; by trial α and β, respectively, $= 14°44'$ and $30°\ 16'$.

2. $D = \frac{N}{P_n \cos \alpha} = \frac{73}{8 \times 0.96712} = 9.435$ inches.

3. $d = \frac{N}{P_n \cos \beta} = \frac{73}{8 \times 0.86369} = 10.565$ inches.

4. $O = D + \frac{2}{P_n} = 9.435 + \frac{2}{8} = 9.685$ inches.

5. $o = d + \frac{2}{P_n} = 10.565 + \frac{2}{8} = 10.815$ inches.

6. $T = N \div \cos^3 \alpha = 73 \div 0.904 = 81$ teeth.

7. $t = N \div \cos^3 \beta = 73 \div 0.645 = 113$ teeth.

8. $L = \pi D \cot \alpha = \pi \times 9.435 \times 3.803 = 112.72$ inches.

9. $l = \pi d \cot \beta = \pi \times 10.565 \times 1.714 = 56.889$ inches.

11. Shafts at a 45-degree Angle, Ratio Unequal and Center Distance Approximate. — A formula for finding the number of teeth in the small gear is found from the equation:

$$\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \cos \beta} = 2\,C$$

by solving for the value of n, the relation of N to n being:

$$R = \frac{N}{n}, \text{ or } N = Rn.$$

Then multiplying by $P_n \cos \alpha \cos \beta$, we have:

$$N \cos \beta + n \cos \alpha = 2\,CP_n \cos \alpha \cos \beta$$

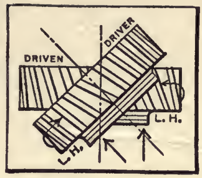
and substituting Rn for N:

$$n = \frac{2\,CP_n \cos \alpha \cos \beta}{R \cos \beta + \cos \alpha}$$

After finding the number of teeth N from the relation $Rn = N$, the pitch diameters are found in the manner previously described. In Part (*a*) are given the formulas to be used when both spiral angles are $22\frac{1}{2}$ degrees. The constants were found from the numerical values of the functions in the formulas of Part (*b*), which latter formulas are used for unequal spiral angles in the two gears.

Formulas, Case 11

The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = center distance.
3. P_n = normal pitch (pitch of cutter).
4. R = ratio of gear to pinion, $N \div n$.
5. α = angle of spiral on gear.
6. β = angle of spiral on pinion.
7. n = number of teeth in pinion nearest $\dfrac{2\, C_a P_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha}$
8. N = number of teeth in gear = Rn.

To find:

(*a*) When $\alpha = \beta = 22\frac{1}{2}$ degrees.

1. D = pitch diameter of gear $= \dfrac{N}{0.9239\, P_n}$
2. d = pitch diameter of pinion $= \dfrac{n}{0.9239\, P_n}$
3. O = outside diameter of gear $= D + \dfrac{2}{P_n}$
4. o = outside diameter of pinion $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter for gear $= N \div 0.788$.
6. t = number of teeth marked on cutter for pinion $= n \div 0.788$.
7. L = lead of spiral on gear = $7.584\, D$.
8. l = lead of spiral on pinion = $7.584\, d$.
9. C = actual center distance $= \dfrac{D + d}{2}$

(*b*) When α and β are any angles.

1. D = pitch diameter of gear $= \dfrac{N}{P_n \cos\alpha}$

2. d = pitch diameter of pinion $= \frac{n}{P_n \cos\beta}$

3. O = outside diameter of gear $= D + \frac{2}{P_n}$

4. o = outside diameter of pinion $= d + \frac{2}{P_n}$

5. T = number of teeth marked on cutter for gear $= N \div \cos^3\alpha$.

6. t = number of teeth marked on cutter for pinion $= n \div \cos^3\beta$.

7. L = lead of spiral on gear $= \pi D \cot\alpha$.

8. l = lead of spiral on pinion $= \pi d \cot\beta$.

9. C = actual center distance $= \frac{D+d}{2}$

Example

Given or assumed:

1. See illustration.
2. $C = 12$ inches.
3. $P_n = 6$.
4. $R = 3$.
5. $\alpha = 20$ deg.
6. $\beta = 25$ deg.
7. $$n = \frac{2CP_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha} = \frac{2 \times 12 \times 6 \times 0.93969 \times 0.90631}{(3 \times 0.90631) + 0.93969}$$ $= 34$ teeth, approx.
8. $N = Rn = 3 \times 34 = 102$ teeth.

To find:

1. $D = \frac{N}{P_n \cos\alpha} = \frac{102}{6 \times 0.93969} = 18.091$ inches.
2. $d = \frac{n}{P_n \cos\beta} = \frac{34}{6 \times 0.90631} = 6.252$ inches.
3. $O = D + \frac{2}{P_n} = 18.091 + \frac{2}{6} = 18.424$ inches.
4. $o = d + \frac{2}{P_n} = 6.252 + \frac{2}{6} = 6.585$ inches.
5. $T = N \div \cos^3\alpha = 102 \div 0.83 = 123$ teeth.
6. $t = n \div \cos^3\beta = 34 \div 0.744 = 46$ teeth.
7. $L = \pi D \cot\alpha = \pi \times 18.091 \times 2.747 = 156.12$ inches.
8. $l = \pi d \cot\beta = \pi \times 6.252 \times 2.145 = 42.13$ inches.
9. $C = \frac{D+d}{2} = \frac{18.091 + 6.252}{2} = 12.1715$ inches.

12. Shafts at a 45-degree Angle, Ratio Unequal and Center Distance Exact. — This case could be used under the same conditions spoken of in Case (8). The number of teeth is found exactly as in Case (11), after which the exact spiral angles are found by trial from the equation:

$$R \sec \alpha + \sec \beta = \frac{2\,CP_n}{n}$$

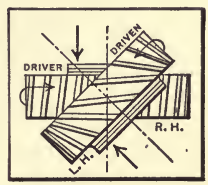
which, in turn, is found from the equations

$$\frac{N}{P_n \cos \alpha} + \frac{n}{P_n \cos \beta} = 2\,C, \text{ and } N = Rn$$

in the same manner as before. When using these equations for finding spiral angles, the trigonometrical tables used must give values to minutes in order to insure accuracy.

Formulas, Case 12

The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. R = ratio of large to small gear $= N \div n$.
4. α_a = approximate spiral angle of large gear.
5. β_a = approximate spiral angle of small gear.
6. C = center distance.
7. n = number of teeth in small gear nearest $\dfrac{2\,CP_n \cos \alpha_a \cos \beta_a}{R \cos \beta_a + \cos \alpha_a}$
8. N = number of teeth in large gear $= Rn$.

To find:

1. α and β, exact spiral angles, by trial from $R \sec \alpha + \sec \beta = \dfrac{2\,CP_n}{n}$

2. D = pitch diameter of large gear $= \dfrac{N}{P_n \cos \alpha}$

3. d = pitch diameter of small gear $= \dfrac{n}{P_n \cos \beta}$

4. O = outside diameter of large gear $= D + \dfrac{2}{P_n}$

5. o = outside diameter of small gear $= d + \dfrac{2}{P_n}$

6. T = number of teeth marked on cutter for large gear $= N \div \cos^3 \alpha$.

7. t = number of teeth marked on cutter for small gear $= n \div \cos^3 \beta$.

8. L = lead of spiral for large gear $= \pi D \cot \alpha$.

9. l = lead of spiral for small gear $= \pi d \cot \beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 4$. 3. $R = 4$.
4. $\alpha_a = 50$ degrees. 5. $\beta_a = 5$ degrees. 6. $C = 30$ inches.
7. $n = \dfrac{2CP_n \cos\alpha_a \cos\beta_a}{R\cos\beta_a + \cos\alpha_a} = \dfrac{2 \times 30 \times 4 \times 0.643 \times 0.996}{(4 \times 0.996) + 0.643} = 33$ teeth.
8. $N = Rn = 4 \times 33 = 132$ teeth.

To find:

1. α and β from $R \sec\alpha + \sec\beta = \dfrac{2\,CP_n}{n} = \dfrac{2 \times 30 \times 4}{33} = 7.273$;

   by trial $\alpha = 50°\ 21'$, and $\beta = 5°\ 21'$.

2. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{132}{4 \times 0.63810} = 51.716$ inches.

3. $d = \dfrac{n}{P_n \cos\beta} = \dfrac{33}{4 \times 0.99564} = 8.286$ inches.

4. $O = D + \dfrac{2}{P_n} = 51.716 + \dfrac{2}{4} = 52.216$ inches.

5. $o = d + \dfrac{2}{P_n} = 8.286 + \dfrac{2}{4} = 8.786$ inches.

6. $T = N \div \cos^3\alpha = 132 \div 0.26 = 508$ teeth.

7. $t = n \div \cos^3\beta = 33 \div 0.987 = 33$ teeth.

8. $L = \pi D \cot\alpha = \pi \times 51.716 \times 0.82874 = 134.6$ inches.

9. $l = \pi d \cot\beta = \pi \times 8.286 \times 10.678 = 278$ inches.

Spiral Gears with Shafts at Any Angle. — When designing spiral gears with shafts at an angle other than 90 degrees to each other, it is of considerable advantage to draw the outline of one gear on a piece of drawing paper tacked to the board, and the outline of the other on a piece of tracing paper, as indicated in the accompanying engraving, Fig. 34. In this way the gear drawn on the tracing paper can be moved about to the correct angle with relation to the gear beneath, and the conditions of

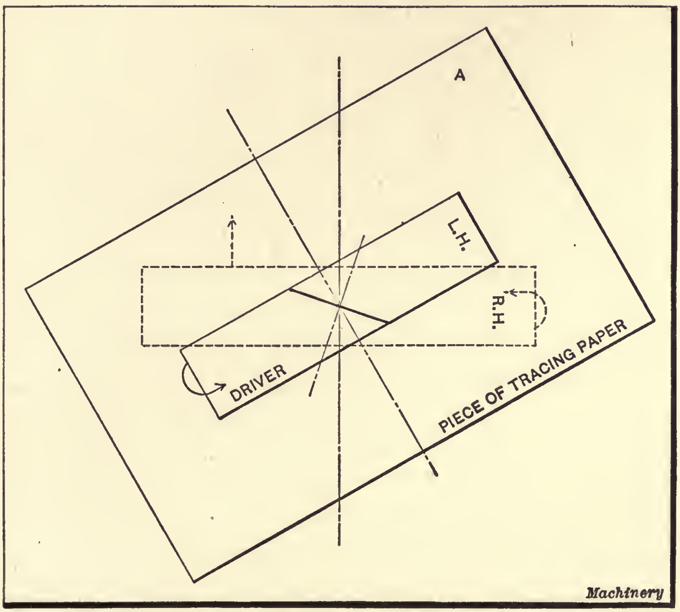


Fig. 34. Method of using Tracing Paper for Spiral Gear Problems

thrust, direction of rotation and hand of spiral can be more easily determined. The thrust diagrams, Figs. 13 to 28, apply also to the gears at present dealt with. With the shafts at any given angle, the sum of the spiral angles of the two gears must equal the angle between the shafts, and the spiral must be of the same hand in both gears, if each spiral angle is less than the shaft angle; but if the spiral angle of one of the gears is greater than the shaft angle, then the *difference* between the spiral angles of the two gears will be equal to the shaft angle, and the gears will be of opposite hand.

Detailed explanation of the derivation of the formulas in the following four cases is unnecessary, as these are arrived at in a manner similar to that referred to in the previous cases. It may be mentioned, however, with relation to Case (15) that the formula for the number of teeth in the smaller gear, in the case when both have the same spiral angles, is found by substituting $\cos \alpha$ for $\cos \beta$ in the formula:

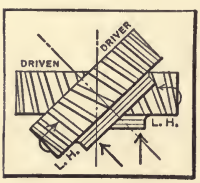
$$\frac{2\,C_a P_n \cos \alpha \cos \beta}{R \cos \beta + \cos \alpha}$$

from which we get the number of teeth in the pinion:

$$\frac{2\,C_a P_n \cos \alpha \cos \alpha}{R \cos \alpha + \cos \alpha} = \frac{C_a P_n \cos \alpha}{R + 1}$$

In Case (16) it sometimes happens, after the exact spiral angles α and β, and the corresponding pitch diameters have been determined, that the center distance does not come exactly as required, within a few thousandths inch. Theoretically it would then be necessary to alter the spiral angles found from one-quarter to one-half a minute, in order that the center distance may figure out correctly. However, this refinement is of doubtful practical value, as it would be impossible to set the machine on which the gears are to be cut to such a minute sub-division of a degree.

13. Shafts at any Angle, Ratio Equal, Center Distance Approximate. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. α = angle of spiral of one gear.

5. β = angle of spiral of other gear.

6. N = number of teeth nearest $\dfrac{2\,C_a P_n \cos\alpha \cos\beta}{\cos\alpha + \cos\beta}$

To find:

1. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos\alpha}$

2. d = pitch diameter of other gear $= \dfrac{N}{P_n \cos\beta}$

3. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$

4. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$

5. T = number of teeth marked on cutter for one gear $= N \div \cos^3\alpha$.

6. t = number of teeth marked on cutter for other gear $= N \div \cos^3\beta$.

7. L = lead of spiral for one gear $= \pi D \cot\alpha$.

8. l = lead of spiral for other gear $= \pi d \cot\beta$.

9. C = actual center distance $= \dfrac{D + d}{2}$

Example

Given or assumed (angle of shafts, 30 degrees):

1. See illustration. 2. $C_a = 5$ inches. 3. $P_n = 10$.
4. $\alpha = 20$ degrees. 5. $\beta = 10$ degrees. 6. $N = 48$.

To find:

1. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{48}{10 \times 0.9397} = 5.108$ inches.

2. $d = \dfrac{N}{P_n \cos\beta} = \dfrac{48}{10 \times 0.9848} = 4.874$ inches.

3. $O = D + \dfrac{2}{P_n} = 5.108 + \dfrac{2}{10} = 5.308$ inches.

4. $o = d + \dfrac{2}{P_n} = 4.874 + \dfrac{2}{10} = 5.074$ inches.

5. $T = N \div \cos^3\alpha = 48 \div 0.83 = 58$ teeth.

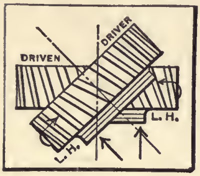
6. $t = N \div \cos^3\beta = 48 \div 0.96 = 50$ teeth.

7. $L = \pi D \cot\alpha = \pi \times 5.108 \times 2.747 = 44.08$ inches.

8. $l = \pi d \cot \beta = \pi \times 4.874 \times 5.671 = 86.84$ inches.

9. $C = \frac{D + d}{2} = \frac{5.108 + 4.874}{2} = 4.991$ inches.

14. Shafts at Any Angle, Ratio Equal, Center Distance Exact.—The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C = center distance.
3. P_n = normal pitch (pitch of cutter).
4. α_a = approximate spiral angle of one gear.
5. β_a = approximate spiral angle of other gear.
6. N = number of teeth nearest $\frac{2\,CP_n \cos \alpha_a \cos \beta_a}{\cos \alpha_a + \cos \beta_a}$

To find:

1. α and β = exact spiral angles, found by trial from $\sec \alpha + \sec \beta = \frac{2\,CP_n}{N}$
2. D = pitch diameter of one gear $= \frac{N}{P_n \cos \alpha}$
3. d = pitch diameter of other gear $= \frac{N}{P_n \cos \beta}$
4. O = outside diameter of one gear $= D + \frac{2}{P_n}$
5. o = outside diameter of other gear $= d + \frac{2}{P_n}$
6. T = number of teeth marked on cutter for one gear $= N \div \cos^3 \alpha$.
7. t = number of teeth marked on cutter for other gear $= N \div \cos^3 \beta$.
8. L = lead of spiral for one gear $= \pi D \cot \alpha$.
9. l = lead of spiral for other gear $= \pi d \cot \beta$.

Example

Given or assumed (angle of shafts, 50 degrees):

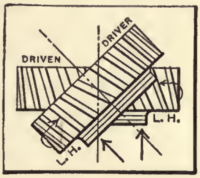
1. See illustration. 2. $C = 10$ inches. 3. $P_n = 10$.
4. $\alpha_a = 20$ deg. 5. $\beta_a = 30$ deg.
6. $$N = \frac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a} = \frac{2 \times 10 \times 10 \times 0.93969 \times 0.86603}{0.93969 + 0.86603}$$
$$= 90 \text{ teeth.}$$

To find:

1. α and β from $\sec\alpha + \sec\beta = \frac{2\,CP_n}{N} = \frac{2 \times 10 \times 10}{90} = 2.222$;

   by trial α and β, respectively, $= 19° \, 20'$ and $30° \, 40'$.
2. $$D = \frac{N}{P_n \cos\alpha} = \frac{90}{10 \times 0.94361} = 9.537 \text{ inches.}$$
3. $$d = \frac{N}{P_n \cos\beta} = \frac{90}{10 \times 0.86015} = 10.463 \text{ inches.}$$
4. $$O = D + \frac{2}{P_n} = 9.537 + \frac{2}{10} = 9.737 \text{ inches.}$$
5. $$o = d + \frac{2}{P_n} = 10.463 + \frac{2}{10} = 10.663 \text{ inches.}$$
6. $T = N \div \cos^3\alpha = 90 \div 0.84 = 107$ teeth.
7. $t = N \div \cos^3\beta = 90 \div 0.64 = 141$ teeth.
8. $L = \pi D \cot\alpha = \pi \times 9.537 \times 2.85 = 85.39$ inches.
9. $l = \pi d \cot\beta = \pi \times 10.463 \times 1.686 = 55.42$ inches.

15. Shafts at Any Angle, Ratio Unequal, Center Distance Approximate. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = center distance.
3. P_n = normal pitch (pitch of cutter).

4. R = ratio of gear to pinion = $N \div n$.
5. α = angle of spiral on gear.
6. β = angle of spiral on pinion.
7. n = number of teeth in pinion nearest $\frac{2\,C_a P_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha}$ for any angle, and $\frac{2\,C_a P_n \cos\alpha}{R + 1}$ when both angles are equal.
8. N = number of teeth in gear = Rn.

To find:

1. D = pitch diameter of gear = $\frac{N}{P_n \cos\alpha}$
2. d = pitch diameter of pinion = $\frac{n}{P_n \cos\beta}$
3. O = outside diameter of gear = $D + \frac{2}{P_n}$
4. o = outside diameter of pinion = $d + \frac{2}{P_n}$
5. T = number of teeth marked on cutter for gear = $N \div \cos^3\alpha$.
6. t = number of teeth marked on cutter for pinion = $n \div \cos^3\beta$.
7. L = lead of spiral on gear = $\pi D \cot\alpha$.
8. l = lead of spiral on pinion = $\pi d \cot\beta$.
9. C = actual center distance = $\frac{D+d}{2}$

Example

Given or assumed (angle of shafts, 60 degrees):

1. See illustration. 2. $C_a = 12$ inches. 3. $P_n = 8$.
4. $R = 4$. 5. $\alpha = 30$ degrees. 6. $\beta = 30$ degs.
7. $n = \frac{2\,C_a P_n \cos\alpha}{R + 1} = \frac{2 \times 12 \times 8 \times 0.86603}{4 + 1} = 33$ teeth.
8. $N = 4 \times 33 = 132$ teeth.

To find:

1. $D = \frac{N}{P_n \cos\alpha} = \frac{132}{8 \times 0.86603} = 19.052$ inches.
2. $d = \frac{n}{P_n \cos\beta} = \frac{33}{8 \times 0.86603} = 4.763$ inches.
3. $O = D + \frac{2}{P_n} = 19.052 + \frac{2}{8} = 19.302$ inches.

4. $o = d + \frac{2}{P_n} = 4.763 + \frac{2}{8} = 5.013$ inches.

5. $T = N \div \cos^3 \alpha = 132 \div 0.65 = 203$ teeth.

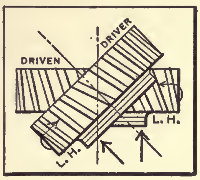
6. $t = n \div \cos^3 \beta = 33 \div 0.65 = 51$ teeth.

7. $L = \pi D \cot \alpha = \pi \times 19.052 \times 1.732 = 103.66$ inches.

8. $l = \pi d \cot \beta = \pi \times 4.763 \times 1.732 = 25.92$ inches.

9. $C = \frac{D + d}{2} = \frac{19.052 + 4.763}{2} = 11.9075$ inches.

16. Shafts at Any Angle, Ratio Unequal, Center Distance Exact. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C = center distance.
3. P_n = normal pitch (pitch of cutter).
4. α_a = approximate spiral angle of gear.
5. β_a = approximate spiral angle of pinion.
6. R = ratio of gear to pinion = $N \div n$.
7. n = number of teeth in pinion nearest $\frac{2\,CP_n \cos \alpha_a \cos \beta_a}{R \cos \beta_a + \cos \alpha_a}$
8. N = number of teeth in gear = Rn.

To find:

1. α and β, exact spiral angles, found by trial from $R \sec \alpha + \sec \beta = \frac{2\,CP_n}{n}$
2. D = pitch diameter of gear = $\frac{N}{P_n \cos \alpha}$
3. d = pitch diameter of pinion = $\frac{n}{P_n \cos \beta}$
4. O = outside diameter of gear = $D + \frac{2}{P_n}$

5. o = outside diameter of pinion $= d + \frac{2}{P_n}$

6. T = number of teeth marked on cutter for gear $= N \div \cos^3 \alpha$.

7. t = number of teeth marked on cutter for pinion $= n \div \cos^3 \beta$.

8. L = lead of spiral on gear $= \pi D \cot \alpha$.

9. l = lead of spiral on pinion $= \pi d \cot \beta$.

Example

Given or assumed (angle of shafts, 60 degrees):

1. See illustration. 2. $C = 40$ inches. 3. $P_n = 4$.
4. $\alpha_a = 20$ degrees. 5. $\beta_a = 40$ degrees. 6. $R = 3$.
7. $n = \frac{2\,CP_n \cos \alpha_a \cos \beta_a}{R \cos \beta_a + \cos \alpha_a} = \frac{2 \times 40 \times 4 \times 0.9397 \times 0.766}{(3 \times 0.766) + 0.9397}$
$= 71$ teeth.
8. $N = Rn = 3 \times 71 = 213$ teeth.

To find:

1. α and β from $R \sec \alpha + \sec \beta = \frac{2\,CP_n}{n} = \frac{2 \times 40 \times 4}{71} = 4.507$;
by trial $\alpha = 22° \; 24' \; 30''$ and $\beta = 37° \; 35' \; 30''$.
2. $D = \frac{N}{P_n \cos \alpha} = \frac{213}{4 \times 0.92449} = 57.599$ inches.
3. $d = \frac{n}{P_n \cos \beta} = \frac{71}{4 \times 0.79238} = 22.401$ inches.
4. $O = D + \frac{2}{P_n} = 57.599 + \frac{2}{4} = 58.099$ inches.
5. $o = d + \frac{2}{P_n} = 22.401 + \frac{2}{4} = 22.901$ inches.
6. $T = N \div \cos^3 \alpha = 213 \div 0.79 = 270$ teeth.
7. $t = n \div \cos^3 \beta = 71 \div 0.497 = 143$ teeth.
8. $L = \pi D \cot \alpha = \pi \times 57.599 \times 2.4252 = 438.8$ inches.
9. $l = \pi d \cot \beta = \pi \times 22.401 \times 1.2989 = 91.41$ inches.

Special Case of Spiral Gear Design. — The following method is used when the distance between the centers of the shafts, the speed ratio and an approximate ratio of the pitch diameters of the gears are given. (Shafts at 90 degrees angle.) In the formulas, let:

D = diameter of driver;
d = diameter of driven gear;
S = speed of driver;
s = speed of driven gear;
P_n = normal diametral pitch;
α = angle of teeth in driver with its axis;
N = number of teeth in driver;
n = number of teeth in driven gear;
C = center distance.

Assume trial values for D and d; then an approximate angle α is derived from the formula:

$$\frac{ds}{DS} = \cot \alpha. \tag{1}$$

Then find by trial the number of teeth for each of the gears which, with the given speed ratio, will most nearly satisfy the equation:

$$2C = \frac{N}{P_n \cos \alpha} + \frac{n}{P_n \sin \alpha} \tag{2}$$

Then make corrections of the angle α until a value is found which exactly satisfies the last equation. This being done, the pitch diameters are:

$$D = \frac{N}{P_n \cos \alpha} \tag{3}$$

$$d = \frac{n}{P_n \sin \alpha} \tag{4}$$

Example. — Find the diameters and angles of teeth of two spiral gears with shafts at right angles; the distance between the centers is $4\frac{1}{8}$ inches, the speed ratio of the driver to the follower is 2 to 1, and the ratio of D to d is about 9 to 8.

Following the method outlined:

$$\frac{ds}{DS} = \frac{8 \times 1}{9 \times 2} = 0.444 = \cot 66^\circ, \text{ approx.}$$

By trial it will be found that 14 and 28 teeth will nearly satisfy Equation (2). Substituting these numbers of teeth and the functions of 66 degrees in this equation, we have:

$$\frac{14}{8 \times 0.4067} + \frac{28}{8 \times 0.9135} = 8.134.$$

Subtracting 8.250 − 8.134 = 0.116.

We see that the angle of 66 degrees introduces an error of 0.116 inch for twice the distance between centers of shafts. This shows that 66 degrees is not exactly the required angle. Trying 66 degrees 50 minutes, we have:

$$\frac{14}{8 \times 0.3934} + \frac{28}{8 \times 0.9194} = 8.254.$$

Subtracting 8.254 − 8.250 = 0.004.

We see that 66 degrees 50 minutes is very close to the required angle, as the error for twice the distance between the centers of the shafts is now only 0.004 inch; trying an angle of 66 degrees 48 minutes, we have:

$$\frac{14}{8 \times 0.3939} + \frac{28}{8 \times 0.9191} = 8.250.$$

The angle of 66 degrees 48 minutes gives exactly the required distance between centers. We can now use this angle in determining the required diameter for the driver and follower by substitution in Equations (3) and (4).

$$D = \frac{N}{P_n \cos 66° \ 48'} = \frac{14}{8 \times 0.3939} = 4.442 \text{ inches.}$$

$$d = \frac{n}{P_n \sin 66° \ 48'} = \frac{28}{8 \times 0.9191} = 3.808 \text{ inches.}$$

By reference to a table of natural functions, we find that sine 66 degrees 48 minutes equals cosine 23 degrees 12 minutes, and this determines the angle of the teeth in the follower as 23 degrees 12 minutes.

CHAPTER III

HERRINGBONE GEARS

Definitions and Types of Herringbone Gears. — One of the objectionable features of spiral gears is the end thrust necessarily produced when these gears are in action. When spiral gears transmit motion between two parallel shafts, this end thrust may be avoided by placing two spiral gears side by side, having teeth cut in opposite directions, as indicated at *A*, in Fig. 1. This type of gearing has been termed "herringbone" gearing. The placing of two spiral gears side by side, keyed to the same shaft, has, however, certain disadvantages, because it is practically impossible, with ordinary means, to so cut the two gears that the two halves will be in perfect mesh at all times, so that each takes one-half the load. From a practical standpoint herringbone gears have, therefore, been less satisfactory than straight-cut spur gears, because until recently no method was devised for producing them with commercial accuracy at a reasonable rate of speed.

In order to avoid using two separate gears, the two sets of spiral teeth have been cut on the same blank, a groove being formed in the center as indicated at *B*, Fig. 1, and teeth cut on each side. One method has also been developed for cutting the two opposite spirals at the same time, as indicated at *C*. Cast gears, of course, can be made in one piece, as indicated at *D*. Within the last decade a method has been developed to a high degree of perfection, by means of which herringbone gears can be cut both rapidly and with commercial accuracy. The principle of this method is indicated at *E*. Herringbone gears made by this method are called Wuest gears, after the inventor. The difference between these gears and those of the ordinary herringbone type is that the teeth of the former, instead of joining at a common apex at the center of the face are stepped or staggered half of the pitch apart, and thus do not meet at all. This arrangement of the teeth does not affect the action of the gears, but facilitates their commercial production.

One type of gear, shown at *F*, is known as the Citroen gear. In this type of gear the end thrust has been avoided by making the teeth of a "wavy" form — a multiple herringbone type.

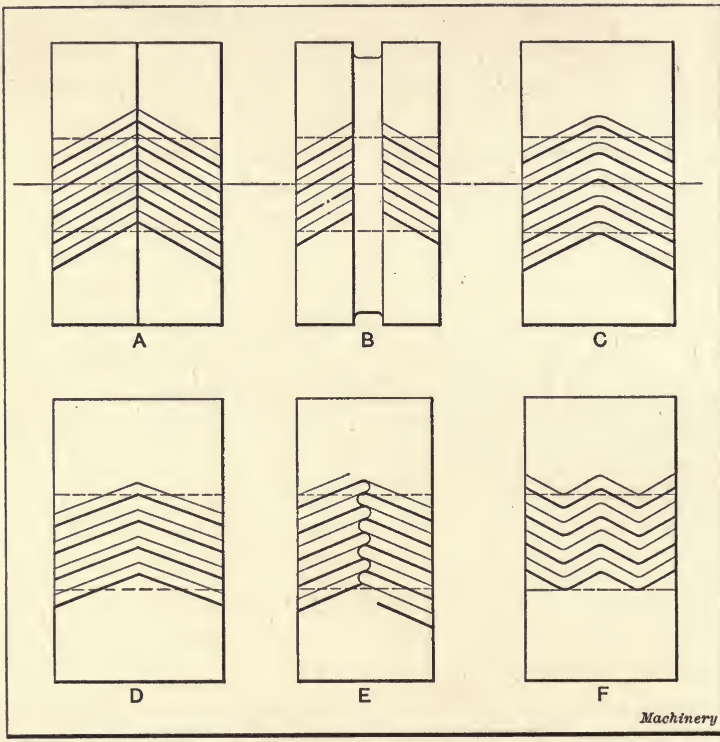


Fig. 1. Diagrammatical Views of Different Types of Herringbone Gears

The difficulties of producing teeth of this kind are obvious and the ordinary herringbone gear is, without question, the type which meets the requirements better than any other method for accomplishing the same result.

Cost of Herringbone Gears. — The herringbone gear as commonly made would cost about twice as much as an equivalent spur gear and is somewhat difficult to cut so that each gear will carry half the load. These drawbacks have overshadowed its advantages for ordinary uses, and as a result its use is almost unknown in the general run of machinery. The advent of the hobbing process of cutting helical gears of the Wuest design, which can be cut very cheaply, has made them available in practically every case, as far as cost is concerned, where spur gears are used. The advantages of herringbone gears are such that machine designers cannot afford to neglect them. The information given in the following is mainly from a paper presented before the American Society of Mechanical Engineers by Mr. Percy C. Day.

Requirements of Power Transmitting Mediums. — The utilization of power constantly calls for means to transmit rotary motion from one axis to another. While there are many ways in which such transmissions may be produced, the merits of all of them must be judged from the following standards: (*a*) reliability and freedom from wear and tear; (*b*) economy of outlay; (*c*) mechanical efficiency; (*d*) compactness; (*e*) evenness of transmission, absence of shock, jar or vibration; (*f*) absence of noise.

Action of Spur Gearing. — The aim of all designers of gearing is to transmit rotary motion from one axis to another in a perfectly even manner without variation of angular velocity. Let us consider the action of a straight spur pinion driving a gear. There are three distinct phases of engagement:

First phase: The root of the pinion tooth engages the point of the gear tooth.

Second phase: The teeth are engaged near the pitch line.

Third phase: The point of the pinion tooth engages the root of the gear tooth.

Let us assume that the teeth are accurately cut to involute form, so that if the pinion moves with even angular velocity it will produce corresponding evenness of motion in the gear; and also that the pinion has sufficient teeth to allow the engagement of successive teeth to overlap. At the beginning of the first phase, while the load is carried near the point of the gear tooth, that tooth is subjected to a maximum bending stress along its whole length. The portion of the pinion tooth near the root is sliding over the outer portion of the gear tooth; that is to say, two metallic surfaces of small area are sliding under heavy compression.

The action during the second phase more nearly approaches ideal conditions. The teeth are engaged near their respective pitch lines and very little sliding takes place. During the third and final phase the pinion tooth is subjected to a maximum bending stress, while the tooth surfaces again slide over each other, this time with the outer portion of the pinion tooth engaging the gear tooth near its root. The point to be noted is that while those portions of the mating teeth which are near the pitch lines transmit the load with rolling contact, those which are more remote have to transmit the same load with sliding contact. The inevitable result is that the points and roots of all the teeth tend to wear away more rapidly than the portions near the pitch lines.

It may be suggested that the sliding action can be eliminated by shortening the teeth so that they engage only during the phase of rolling contact. This has been tried with a certain measure of success in the stub-toothed gear, but it cannot be carried far enough without curtailing the arc of contact so that continuity of engagement is lost.

Distortions of gear teeth of involute form, whether due to inaccurate cutting or subsequent wear, give rise to all kinds of trouble. The average angular velocity may be uniform, and yet the passage of each pinion tooth through its brief engagement with the mating gear may be accompanied by successive retardation and acceleration which, though small in itself, takes place in such a short interval of time that it may cause stresses many times greater than the average working load on the teeth. These internal stresses are very difficult to deal with, because they are indeterminate. They cause noise, vibration, crystallization and fracture.

Action of Herringbone Gears. — Herringbone gears completely overcome all these difficulties, but only when they are accurately cut. If we take two exactly similar pinions with straight teeth and place them side by side on one shaft, with the teeth of one pinion set opposite the spaces of the other, then we have what is known as a stepped-tooth pinion. If this pinion is meshed with a composite gear made up in a similar manner, the action is modified so that there are always two phases of engagement taking place simultaneously. Such gears are commonly used for rolling mill work, because they stand up to heavy shocks better than the plain type. Still better action can be secured by assembling a number of narrow pinions with the last of the series one pitch in advance of the first and the others advanced by equal angular increments. As a practical proposition, however, gears made on these lines would be costly and difficult to produce.

The helical gear is the logical outcome of the stepped gear carried to its limit, and built up from infinitely thin laminations. Since the steps have merged into a helix, there must be a normal component of the tangential pressure on the teeth, producing end thrust on the shafts. To obviate end thrust the helical teeth are made right-hand on one side and left-hand on the other. Such gears, as already stated, are known as herringbone gears.

The fundamental principle of the action of herringbone teeth lies in the circumstance that *all phases of engagement take place simultaneously.* This holds good for every position of pinion and gear, provided only that the relationship between pitch, face width and spiral angle is such as will insure a complete overlap of engagement. Since all phases of engagement occur together, it follows that the load is partly carried by tooth surfaces in sliding contact and partly by surfaces in rolling contact. The result is curious and interesting.

Those portions of the teeth farthest from the pitch line, which engage with sliding action, tend to wear away more rapidly than the portions nearest the pitch line; but the pitch line portion is always carrying part of the load, and the effect of wear on the ends of the teeth merely tends to throw more load on the center portions; in other words *there is a tendency to concentrate the load near the pitch lines.* The ends of the teeth, instead of wearing away to an ever-increasing extent from their original involute form, are relieved of some of the load from the moment that wear commences to take place. As soon as the load on these ends has been partially relieved and transferred to the middle portion, the wear becomes equalized all over the teeth and they do not tend to distort further from their original shape.

It is quite clear that an unmeasurable amount of wear on the tooth ends will be sufficient to relieve them of all the load, so that the distortion from the original form will be practically nothing. The minute extra wear that does take place at the ends is only the amount necessary to transfer a certain proportion of the load near the pitch lines, so that the wear is equalized all over the surface of the teeth, those portions in sliding contact carrying less than those in rolling contact.

Advantages Gained by Herringbone Gears. — As the teeth keep their involute form, motion is transmitted from the pinion to the gear in an even manner, without jar, shock or vibration. Although herringbone teeth may not be intrinsically stronger than straight teeth, the elimination of shock renders them capable of transmitting heavier loads. Since all phases of engagement occur simultaneously, the transference of the load from one pinion tooth to the next takes place gradually instead of suddenly. This is the second principle of herringbone gearing, and may be termed *continuity of action.*

In straight gears the continuity of action is a function of the number of teeth in the pinion. In herringbone gears continuity depends on the relationship between the face width and the number of teeth in the pinion. Pinions with as few as five teeth have been used with success by merely increasing the face width to suit such extreme conditions. This feature, which is peculiar to herringbone gears, has made practical the adoption of extremely high ratios of reduction hitherto considered impossible.

The third principle of herringbone gearing is that the bending stress on the teeth does not fluctuate from maximum to minimum as in straight gears, but remains always near the mean value. This feature is of special importance in rolling-mill driving and work of a similar nature.

To summarize the foregoing statements: The action of herringbone gears is continuous and smooth; there is no shock of transference from tooth to tooth; the teeth do not wear out of shape; the bending action of the load on the teeth is less than with straight gearing and does not fluctuate to anything like the same extent; the gears work silently and without vibration; back-lash is absent; friction and mechanical losses are reduced to a minimum; herringbone gears can be used for higher ratios and greater velocities than any other kind.

Production of Herringbone Gears. — Herringbone gears may be produced in a variety of ways which differ from each other as widely as the character of the product. Until a few years ago all gears of this type were molded. The limitations of molded gearing are analogous to those which would be experienced if a journal were run in a molded bearing. Just as the bearing would touch the shaft only in spots, so molded gears fail to give the intimate contact all along the teeth which is necessary to secure the realization of true helical gear action. It is obvious that if the teeth touch only in a few high places, they will be subjected to all the evils of shock, stress and inequality of motion which it is desired to avoid. If the gears are particularly well molded, some mitigation of these evils may be expected when they become well worn, but the initial wear is accompanied by a departure from the correct tooth shape.

For slow speeds a well-molded helical gear is no better than a straight gear with cut teeth, and for high speeds it is not as good. The natural smoothness of helical action does no more than compensate for the inaccuracies of tooth form and spacing. The modern herringbone gear must have cut teeth if its advantages are to become realized.

One- and Two-piece Types. — Cut herringbone gears may be broadly divided into two classes, two-piece and one-piece gears. The difficulty in the way of cutting double helical teeth in a single blank gave rise to the two-piece variety. The same methods of cutting may be used for both kinds. The disadvantages of the two-piece type are obvious. There is the expense of two complete gears, the difficulty of assembling so that the gears be in accurate register with each other, and the necessity for very thorough fastenings if they are to perform hard service without getting out of register. High ratios are not within the scope of the built-up gear, because the pinions must be assembled on a separate shaft and the pitch line must be far enough from the surface of the shaft to allow room for the necessary bolts or rivets used in fastening the two portions together. The one-piece pinion, however, may be cut solid with its shaft, so that its pitch diameter need be but very little larger than the latter.

The methods of cutting helical gears may be divided into four classes: (*a*) milling by formed disk cutters; (*b*) milling by end-mills; (*c*) generating by shaping or planing methods; (*d*) generating by hobs.

Milling by Ordinary Disk Cutters. — Milling by formed disk cutters is unsatisfactory, because, in addition to the usual errors of step-by-step division, there is the difficulty of making the cutters to the *normal* tooth shape with sufficient accuracy to insure correct circumferential shape for the gears cut. This difficulty is increased by reason of the fact that a disk cutter cannot cut its own shape in a spiral groove. Let it be noted that the cutters must be formed empirically, that their number must be very large to meet the requirements of a general gear business, and that the accuracy of each gear turned out depends on the combined efforts of the toolmaker and draftsman who produced the cutter. Worst of all, two different cutters must be used for a gear and pinion. This method will produce indifferent herringbone gears whether they are built up with teeth in register or made in one piece with staggered teeth.

Milling by End-mills. — The use of end-mills is open to all the objections to disk cutters, with the single exception that the cutter does leave a fair approximation to its own shape in the groove which it cuts; but the end-mill has a number of disadvantages peculiar to itself which render it even less efficient than the disk cutter for general work. In the first place it is a small tool with very little wearing surface and no capacity for dissipating the heat generated at its cutting edges. The great variation in diameter between point and base renders it difficult to arrive at a cutting speed which will satisfy the conditions at both ends of the cut. The mills quickly become clogged with cuttings, overheat and burn. To complete one fair-sized gear by the end-mill process requires quite a number of cutters. This not only makes the expense heavy, but must necessarily result in an inaccurate gear.

Every cutter used must be formed to gage and hardened. After being hardened, it will run a trifle out of true, in most cases, thereby cutting a shape different from that for which it was designed. In end-milled gears it is not merely a case of getting accurate conjugate tooth shapes in gear and pinion made with different cutters, but the teeth in a single gear may have a dozen different shapes. The process is so slow that it cannot compete with other methods, quite apart from the doubtful quality of the gears produced.

End-milled herringbone gears are usually made in one piece with the teeth joined at the center. Since the cutter is shaped to the normal pitch, it follows that, in changing over from right- to left-hand helix, it leaves a thick wedge in the center of the face which must be removed by a subsequent operation. The teeth of end-milled herringbone gears do not bear over the center portion.

Planing the Gear Teeth. — Generating processes of the shaping and planing type, while successful for straight-cut gears of relatively small size, are not used to any extent for large diameters or heavy pitches. The reason for this may be found in the nature of the processes. The gear blank is required to make a quick angular movement after each stroke of the cutting tool and to come to rest again before the next stroke. Such methods are difficult to apply to large gears on account of the inertia of the gear blank and its support and the consequent difficulties of controlling the short intermittent movements. These difficulties are much increased when such methods are applied to cutting helical teeth because the blank must make definite and rapid angular movements during each stroke in addition to the motion between strokes.

The Hobbing Process. — The hobbing process as applied to straight-cut gears has proved very successful, and it is not difficult to understand why this process has sprung into prominence in a comparatively short time. It is essentially a rational process. The shape of the teeth is generated from spiral hobs, the threads of which are cut to a plain rack section. One hob will cut any gear or pinion of one pitch. This feature alone eliminates a great many errors which are characteristic of gears produced by milling methods. The hob revolves continuously while cutting, as does the gear blank. The feed is also continuous. There are no cutting and return strokes, and no intermittent starting and stopping of gear blanks, as in other generating processes. These features do not necessarily insure the production of accurate gears, but they offer greater facilities to the designer for the achievement of the desired result.

The hob is a substantial tool with plenty of wearing and cooling surface, and can be made to meet the demands of rapid production and to last for a long time. The continuous nature of all motions used in hobbing a gear blank enables this process to be used for the production of the heaviest gears. The limit to the size of a hobbing machine is set by the dimensions of the largest gears which are required in sufficient quantities to pay for the investment.

Nevertheless, there are some defects in the hobbing process as applied to the production of straight-cut spur gears. A hob is a worm thread, and as such must have a spiral angle depending on the relationship between the pitch of the thread and the diameter of the hob. A straight-cut gear has no spiral angle, hence the spiral hob must be inclined, more or less, to bring the cutters in line with the tooth spaces to be cut. In order to cut correct teeth, the axis of the hob should be perpendicular to the axis of the gear blank. In such case the hob will generate involute teeth if its threads are cut to the same axial section as the straight-sided parent rack for the required pitch. Since the hob must be inclined to cut a spur gear, the teeth are not generated from the axial or rack section, but from a diagonal section. The axial pitch of a hob for cutting spur gears is not the same as the pitch of the gears which it cuts. The normal pitch of the hob threads must be the same as the gear pitch.

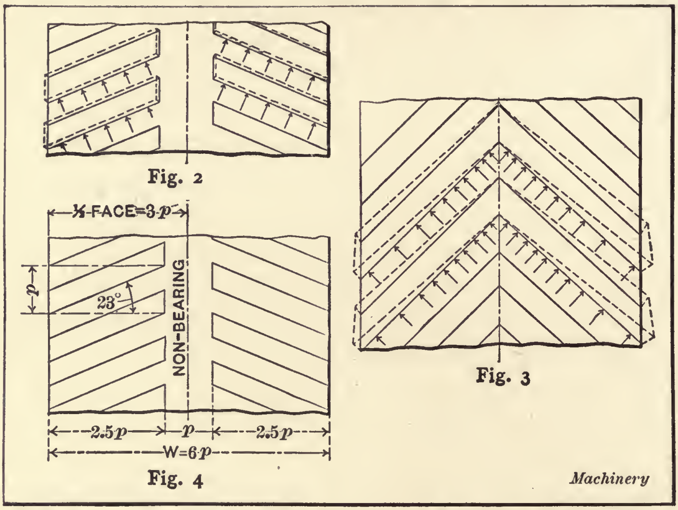
Hobs for cutting straight spur gears are usually made of large diameter to reduce the spiral angle and consequent errors of tooth form to a negligible minimum. As a natural consequence, such hobs have only one thread, while their large diameter requires a slow speed of rotation to keep the cutting speed within proper limits. The effect of this is that the blank revolves very slowly, and a coarse feed must be used to keep up the output.

It is one of the peculiarities of the hob action that only one tooth of the hob puts the finishing touch to the bottom of a tooth space once in each revolution of the gear blank. If the feed is coarse, there will be noticeable feed marks and roughness in the gear teeth produced. A coarse feed used with a hob of large radius throws severe stresses on the hob arbor and its supports.

The necessity of a swivel motion on the hob slide, to enable straight spurs to be cut with hobs of varying spiral angle, compels the use of a hob drive which passes through the pivot. It is almost impossible to design such an arrangement without undesirable restriction in the dimensions of driving gears and shafts combined with excessive overhang of the hob arbor in relation to its supporting slide. The general lack of rigidity about most hobbing machines used for the production of spur gears is traceable to the above causes. Rational critics of the hobbing process have based their objections on these features.

The hobbing process properly applied to the production of herringbone gears has none of the disadvantages incidental to its application to spur gear cutting, which have been shown to lie in the necessity of inclining the hob axis. Since a helical gear and a hob must both have a spiral angle, it is only necessary to make the thread angle of the hob the complement to the corresponding angle of the gear teeth to secure the advantages of perpendicular fixed axes. This principle is of great practical value. Since the hob axis is always perpendicular to the axis of the gear blank, it follows that the teeth are generated from the axial and true rack section of the hob, while the linear pitch of the hob is the same as the circular pitch of the gear which it cuts. The hob axis is fixed and the hob can be supported on a rigid slide with the minimum of overhang. There is no restriction to the size and strength of the gears and shafts used to drive the hob.

Wuest Herringbone Gears. — It has been explained that the teeth of the Wuest gears are so designed that those on the right- and left-hand sides of the gears are stepped half a space apart and do not meet at a common apex at the center of the face, as in the usual type of herringbone gear. It has often been



Figs. 2 to 4. Diagrams showing Tooth Pressures and Angle Necessary for Continuity of Action

argued that the ordinary herringbone tooth is stronger than the Wuest tooth, because the latter lacks the support given by the junction of the teeth at the center. This argument would be sound if gear teeth were ever stressed to anywhere near their breaking point. It has been found in practice that considerations of wear so far outweigh those of mere breaking strength that a gear which is designed to give reasonable service will carry anywhere from ten to twenty times the working load without fracture. A point of vastly greater importance is that the stepped form will wear more evenly under extreme loads than the ordinary type. The reason for this is shown in Figs. 2 and 3. The resultant tooth pressure is always normal to the teeth and tends to bend them apart. The stepped form offers a uniform resistance along its whole length, carrying the load from end to end (Fig. 2). The teeth of ordinary herringbone gears tend to separate more at the sides than near the supported center, causing the load to be concentrated toward the center (Fig. 3).

Interchangeable Herringbone Gears. — Any system of gearing, if it is to be generally applied, must be interchangeable. The variable features of involute spur gear teeth are limited to the pressure angle, addendum and dedendum. In a herringbone gear system, we must have, besides, uniformity of spiral angle and relative position of the right- and left-hand teeth.

The standards which have been adopted for Wuest gears are the result of experience gained in Europe during a period of years. The spiral angle of the teeth is about 23 degrees with the axis. The choice of this angle is controlled by a number of considerations, the most important from the user's standpoint being that the angle must be sufficient to allow the engagement of successive pinion teeth to overlap within a reasonable face width. Once this condition is satisfied, there is no advantage in an increase of spiral angle, while there are disadvantages in the use of steep angles. It was necessary, before choosing a definite spiral angle, to determine what constitutes a reasonable face width for this class of gearing.

Width of Face and Spiral Angle. — Since the nature of the action eliminates shock, it follows that the pitch required for given conditions will be much finer than would be chosen for spur gears. On the other hand, the face width will not be less, because there is as much necessity for wearing surface with one kind of tooth as with the other. Spur gears are usually made with a face width equal to three or four times the pitch. Herringbone gears may conveniently have a face width equal to six times the pitch, not because the width of this type actually is greater, but because the pitch is proportionately less.

Starting with a width equal to six times the pitch, and allowing a width equal to the pitch as the non-bearing portion at the center, there remains two and one-half times the pitch available for the teeth on each side. To insure continuity of engagement under all ordinary conditions, each tooth is inclined so as to cover an advance equal to the pitch within its length. The angle of 23 degrees satisfies this requirement (see Fig. 4). There are a few cases where an angle less than 23 degrees would be sufficient, while a steeper angle is only needed if the available face width has to be unduly restricted. Neither of these extreme conditions should influence the choice of angle for an interchangeable system best adapted for general use.

There are other good reasons why a moderate spiral angle is to be preferred. In all spiral gears the pressure acts in a direc-

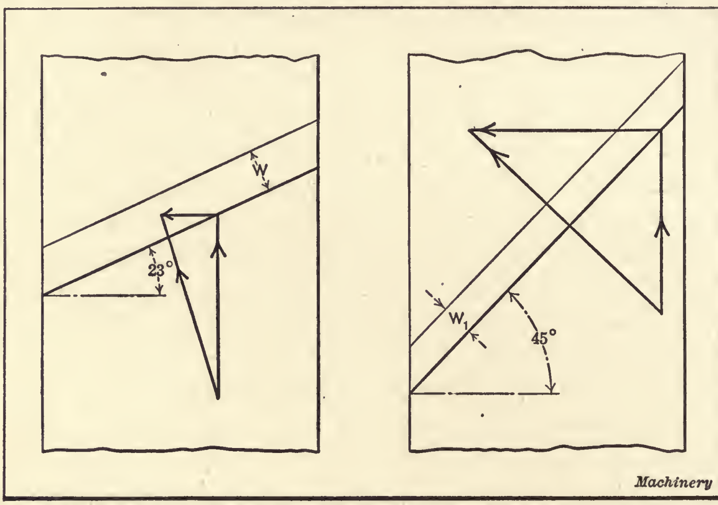


Fig. 5. Diagrams showing Relation between Normal Pressure, Spiral Angle and Available Normal Tooth Section

tion normal to the teeth and is the resultant of the tangential (driving) and axial pressures. The normal pressure becomes greater in proportion to the useful driving pressure as the spiral angle is increased, while the available normal tooth section becomes less (see Fig. 5). When the spiral angle is considerably steeper than the angle of repose for the materials in contact, there is a tendency for the teeth to bind with a wedge action. Herringbone gears with abnormally steep spiral angles show loss of efficiency and increased wear from this cause.

Pressure Angle and Tooth Proportions. — The pressure angle which has been adopted for standard gears is 20 degrees. The teeth are shorter than the usual standards, as indicated by the formulas in the following. These standards of tooth height and pressure angle have been adopted after systematic trials and experience extending over several years of regular manufacture. The high ratios used with these gears call for an average pinion diameter which is less than is used with straight spur gears for similar duty. The teeth are generated by hobs, and the short addendum combined with large pressure angle gives satisfactory tooth shapes, without undercutting of teeth, for small pinions. Pinions with very few teeth are cut on the well-known system of enlarged addendum which is also used for bevel pinions. The teeth are cut to diametral pitch standards, measured circumferentially the same as in ordinary spur gearing.

Diametral Pitch of Herringbone Gears. — Many designers find it difficult to regard herringbone gears as spur gears. They apply the same methods as are used for calculating ordinary spiral gears, and complicate the problem to an unnecessary extent. Spiral gears are usually employed for connecting shafts which are not parallel to each other. Under these conditions the circumferential pitch of gear and pinion may be quite different, but the *normal* pitch of both must be the same.

In herringbone gears, if the spiral angle is made constant there is a definite and fixed relationship between the normal and the circumferential pitch. This is the case with Wuest herringbone gears. It is a great convenience to discard all reference to the normal pitch and treat the gears just like spur gears on the basis of the circumferential pitch. When this is once done, it makes no difference whether the circular or diametral pitch system is used. It is, of course, necessary for the gear cutter to set his calipers to the normal tooth thickness, and if circular cutters or inclined hobs are used they must be designed for the normal pitch the same as in regular spiral gearing; but the designer of machinery involving the use of these gears need not be troubled with any such complications.

Wuest herringbone gears are cut by specially constructed hobs which are used with the hob axis perpendicular to the gear axis. The pitch of each hob, measured along the axis in the same way as the pitch of a screw is measured, is, therefore, the same as the circumferential pitch of the gears which it cuts.

Pitch Diameters and Center Distances. — The question in regard to the use of enlarged pinions is not so easily understood and requires a clear definition of what constitutes the "pitch diameter" and the "pitch" of a gear.

If the center distance and velocity ratio are given, then the true pitch diameters of the gear and pinion are fixed. Now it is well known that involute gears will run satisfactorily when set farther apart than the designed center distance. In other words, the center distance may be varied to a limited extent. This variation of the center distance does not effect either the number of teeth or the velocity ratio, but it alters the pitch. The foregoing arguments lead to the curious conclusion that the pitch of a pair of involute gears has no definite value, but depends on the center distance and velocity ratio. Conversely, if we maintain a fixed center distance and ratio for a given pair of gears, we can cut involute teeth in various ways without altering the pitch.

For instance, if we require a small pinion to mesh with a large gear, we may generate the teeth to standard thickness on their true pitch diameters or we may enlarge the blank diameter of the pinion and reduce that of the gear by a corresponding amount. The teeth will be generated from the same base circles in each case, and the true pitch diameters and pitch will be the same, but the shape of the teeth will be quite different in the two cases. The pinion which is cut on standard lines will probably have badly undercut teeth with consequent weakness and loss of wearing surface. The enlarged pinion, on the contrary, will have teeth with broad bases and unimpaired shape. Since the center distance and velocity ratio have not been altered, the true pitch circles and the pitch remain unchanged; but the change in outside diameters has increased the addendum of the pinion and decreased the addendum of the gear.

There is nothing new in this method, as it has been in use on bevel and worm gears for many years; the most curious thing about it is that it continues to be so little understood by the majority of gear users.

An enlarged pinion will mesh correctly with any gear in its series, whether reduced or not, but if the gear is of standard proportions the center distance will be greater than standard by half the enlargement of the pinion. This applies to all involute gears with generated teeth, no matter whether they are hobbed, shaped or planed. When this method is applied to herringbone gears, the enlargement or reduction of the blank is left entirely out of consideration, and the machine is set to cut the correct spiral angle on the true pitch circle. Given a proper degree of accuracy in the cutting and reasonable care in setting up, such gears are perfectly interchangeable, bear evenly from end to end and do not jam. There is no question of approximation. These methods have been in use for several years, and there are thousands of gears cut in this manner giving satisfactory service.

Dimensions. — The dimensions proposed for an interchangeable system for these gears are, therefore, as follows:

Tooth shape............................ Involute
Pressure angle.......................... 20 degrees
Spiral angle............................ 23 degrees

$$\text{Pitch diameter (20 teeth and over)} = \frac{\text{Number of teeth}}{\text{D.P.}}$$

$$\text{Blank diameter (20 teeth and over)} = \frac{\text{Number of teeth} + 1.6}{\text{D.P.}}$$

$$\text{Pitch diameter (under 20 teeth)} = \frac{0.95 \times \text{No. of teeth} + 1}{\text{D.P.}}$$

$$\text{Blank diameter (under 20 teeth)} = \frac{0.95 \times \text{No. of teeth} + 2.6}{\text{D.P.}}$$

For all herringbone gears, irrespective of number of teeth:

Addendum .... $\frac{0.8}{\text{D.P.}}$

Dedendum .... $\frac{1.0}{\text{D.P.}}$

Full depth .... $\frac{1.8}{\text{D.P.}}$

Working depth .... $\frac{1.6}{\text{D.P.}}$

Standard face width for gears with pinions of not less than 25 teeth, 6 times circular pitch; face widths for high ratio gears with small pinions, 6 to 12 times circular pitch.

When a pinion of less than 20 teeth is used with a standard gear, the center distance must be slightly increased to suit the enlargement of the pinion. If it is desired to keep the center distance to the standard dimensions, the gear diameter may be reduced by the amount of the enlargement given to the pinion. For example, if a pinion of 10 teeth, 5 diametral pitch is to mesh with a gear of 90 teeth at 10-inch centers, then:

$$\text{Pitch diameter of pinion} = \frac{0.95 \times 10 + 1}{5} = 2.1 \text{ inches.}$$

Enlargement over standard pinion = 0.1 inch.

$$\text{Pitch diameter of standard gear} = \frac{90}{5} = 18.0 \text{ inches.}$$

Reduced pitch diameter of gear = 18.0 − 0.1 = 17.9 inches.

$$\text{Center distance} = \frac{17.9 + 2.1}{2} = 10 \text{ inches.}$$

Strength of Herringbone Gears. — In these gears the teeth need not have the same breaking strength as with spur gears because they have not to combat the heavy and indeterminate stresses which arise from inequalities of angular velocity. On the other hand it is necessary to provide against rapid wear. By using a finer pitch, each tooth has less individual wearing surface, but this is more than compensated for by the larger number of teeth in simultaneous contact than is the case with gears of equal diameters but coarser pitch.

In high ratio gears, using pinions of exceptionally small diameter, the pitch is finer than for ordinary ratios, but the face width is extended to give the proper wearing surface.

The important factor in determining the proportions of the teeth is the relationship between pitch line velocity and the permissible specific tooth pressure; in other words, the total tooth pressure divided by the area of all the available simultaneous contact along the teeth. Theoretically, this contact has no area since it should consist of lines without breadth. Actually, an area exists, due to the elastic compression of the teeth in contact, in a similar way in which an area of contact exists between a car wheel and a rail. The area of contact is indeterminate, but the specific tooth pressure is proportional to the driving stress on the teeth.

Horsepower Transmitted. — In order to obtain a simple rule for finding the proper dimensions, the results of experience in the matter of safe working loads under given conditions have been reduced to a relationship between pitch line velocity and the shearing stress on the pitch line thickness of an imaginary straight tooth, assuming only one tooth in engagement at a time. The shearing stress is a measure of the specific tooth pressure, and the relationship referred to affords a convenient means of arriving at reliable dimensions. The curves, Fig. 6, give values of shearing stress K in pounds per square inch on pitch line section of an imaginary single tooth for corresponding pitch line velocities V in feet per minute. The values are entirely empirical, but they are based on the results of extended experience, and lead to dimensions which are safe and reliable. Different curves are given for different materials, and it is necessary to use that curve which corresponds to the lowest grade material of the combination. A table is also provided in which approximate values, taken from the diagram, are given. The dimensions of gears can be derived from the curves in the following manner. In the formulas:

H.P. = horsepower transmitted;
N = revolutions per minute;
D = pitch diameter in inches;
p = circular pitch in inches;
F = total width of face in inches;
V = pitch line velocity in feet per minute;
W = total tooth pressure at pitch line in pounds;
K = stress factor as obtained from table.

Then:

$$V = \frac{3.14\,DN}{12} \qquad W = \frac{\text{H.P.} \times 33{,}000}{V} = \frac{pFK}{2.5}$$

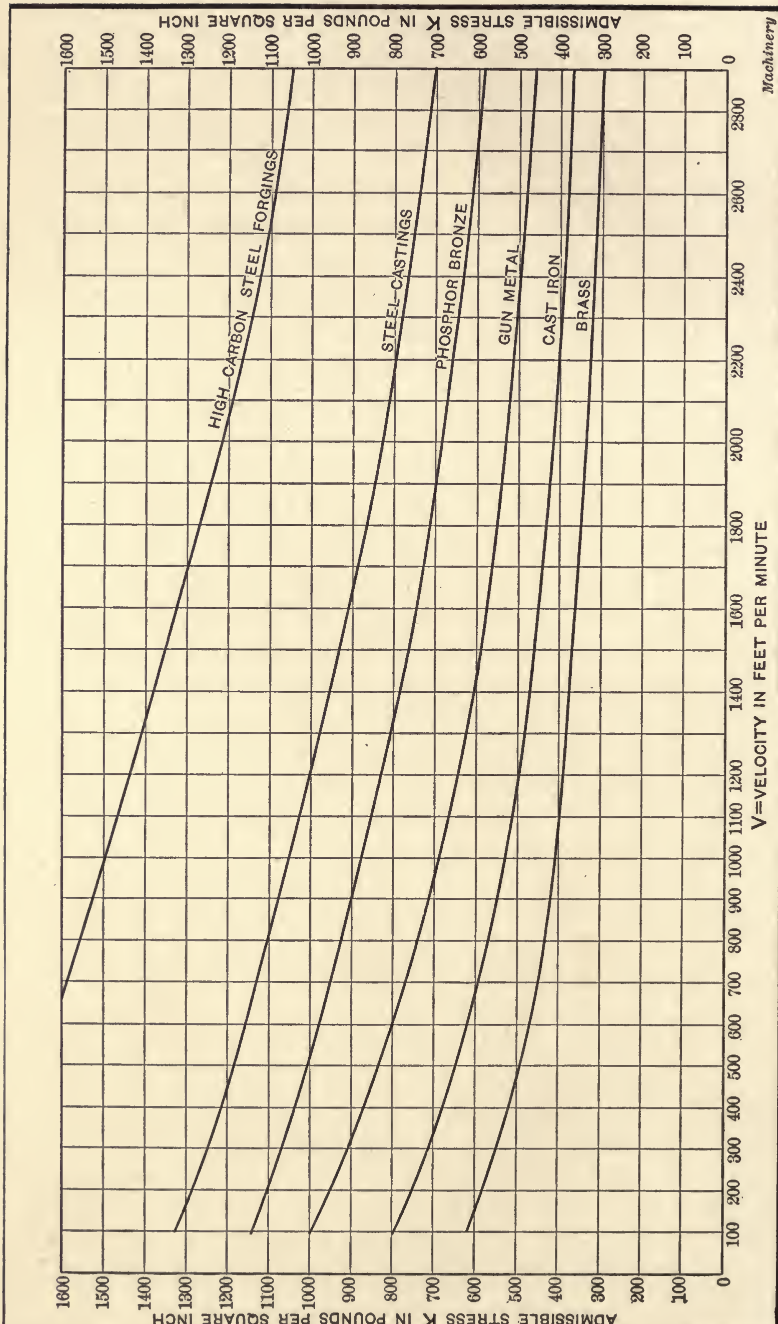


Fig. 6. Values of Permissible Stresses in Herringbone Gear Teeth for Various Metals

In gears with moderate ratio (not exceeding 1 to 6), and face width equivalent to 6 times the circular pitch, make:

$$W = 2.4\, p^2 K, \quad \text{or} \quad p = \sqrt{\frac{W}{2.4\, K}}$$

For higher ratios make $F = Rp$ (where R = ratio of gears) up to a maximum of $F = 10\, p$. The circular pitch for high ratios is found from:

$$p = \sqrt{\frac{2.5\, W}{RK}}$$

When the face width is equivalent to 8 times the circular pitch, $W = 3.2\, p^2 K$, and when the face width is equivalent to 10 times the circular pitch, $W = 4\, p^2 K$.

Table of Safe Shearing Stresses K, in Pounds per Square Inch, for Herringbone Gears

Velocity in Feet per Minute	Factor *K* for					
	Brass	Cast Iron	Gun Metal	Phosphor Bronze	Steel Castings	High-carbon Steel Forgings
100	600	800	1000	1150	1325	1800
200	575	750	950	1100	1275	1750
300	550	700	900	1060	1250	1700
400	525	675	860	1030	1200	1660
500	500	650	830	1000	1175	1630
600	475	625	800	975	1150	1600
800	425	575	750	925	1100	1550
1000	400	525	700	875	1050	1500
1200	380	500	650	825	1000	1450
1500	360	475	600	775	925	1350
1800	350	450	550	725	875	1275
2100	340	425	525	675	825	1200
2400	325	400	500	650	775	1125
3000	300	375	475	600	700	1050

For ordinary service it is safe to use pitch line velocities between 1000 and 2000 feet per minute, with 1500 feet as a fair average. If the pinion is to be fixed to a motor shaft without external support, the diameter must be greater than when it can be supported on both sides. Cast iron is preferable to cast steel for gears of large diameters and moderate power, but the latter will be found more economical for high tooth pressures.

Pinions are usually made from steel forgings of 0.40 to 0.50 per cent carbon. Soft pinions should never be used for herringbone gears.

There are two special cases where the ordinary methods of calculation should not be used. Rolling-mill gears are subjected to stresses which are so far in excess of the average working load that it is necessary to consider carefully the strength of the teeth in regard to possible overloads. Extra high velocity gears, such as are used for steam turbines, require additional wearing surface and are characterized by extreme width of face combined with abnormally fine pitch.

The following is a typical instance of the range of choice in dimensions: A pump which requires 150 horsepower at 50 revolutions per minute is to be driven from a motor at 500 revolutions per minute; the shaft end is $4\frac{1}{2}$ inches in diameter. If the shaft is unsupported, it is not desirable to use a pinion of less than 10 inches diameter. If the shaft is extended to a third bearing a $7\frac{1}{2}$-inch pinion can be used. If the pinion is cut solid on its shaft and coupled to the motor, its diameter can be reduced to 5 inches. The three arrangements work out as follows:

	Material for Gear	*V*	*W*	*K*	Diametral Pitch	Face Width
1. 10 in. and 100 in....	Cast iron	1300	3800	500	2	9½
2. 7½ in. and 75 in....	Cast iron	975	5100	530	2	12
3. 7½ in. and 75 in....	Cast steel	975	5100	1060	2½	7½
4. 5 in. and 50 in....	Cast steel	650	7600	1150	2½	12½

Any of the above gears will do the work satisfactorily; (3) is the most economical, but (2) or (4) would make the least noise. If a gear case is to be provided, then (4) will give the most economical combination.

The foregoing data can be used for finding the required dimensions of herringbone gears for all general applications. In most cases it is sufficient to calculate the tooth pressure from the average working load. When the maximum load is very far in excess of the average, it is usual to take a mean value between the two. Gears for electric mine hoists and single-throw pumps fall within this category. Machine tools, when driven from variable-speed motors, are required to perform maximum duty at minimum speed only for short periods at long intervals. It is sufficient when designing gears for a drive of this kind to reckon with the rated output of the motor at the mean between its maximum and minimum speed.

General Points in Design. — The usual conditions met with in designing any form of gear drive are that power is to be transmitted from a motor spindle or other prime mover running at high speed to a machine which is required to operate at a considerably reduced speed. In selecting a pair of herringbone gears (as in the case of any other form of gearing) the designer naturally selects the smallest size of pinion which can be conveniently adapted to the service for which it is desired. By so doing, the size of the gear is reduced so that the least possible amount of space is required. Another reason for selecting a small pinion and corresponding gear to give the required speed reduction is to reduce the pitch line velocity to a minimum, as a higher velocity means increased wear and objectionable noise. Both the pinion and gear have the same pitch line velocity, and, consequently, the same tooth strength. In the design of herringbone gears a pinion with 21 teeth will be found satisfactory for average conditions, although it is possible to have a pinion with a smaller number of teeth, pinions with as few as 13 teeth having been used with satisfactory results. Where such small-sized pinions are used, however, they are made solid on the shaft.

Tables of Horsepower Transmitted. — The accompanying tables give the horsepower transmitted by herringbone gearing for various pitches in cast iron and cast steel, with pitch line velocities ranging from 400 to 2000 feet per minute. These tables are used for determining the gear which is necessary for transmitting a given power at a specified speed, and are based on the formulas already given for calculating the horsepower transmitted. It is customary to use a pinion of tougher material than the gear, owing to the greater wear to which it is exposed,

Horsepower Transmitted by Herringbone Gears

Velocity at Pitch Circle, Feet per Minute	1½ Diametral Pitch, 21 Teeth, 14-inch Pitch Diameter					1¾ Diametral Pitch, 21 Teeth, 12-inch Pitch Diameter				
	Revolutions per Minute	16-inch Face		20-inch Face		Revolutions per Minute	14½-in Face		18-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	110	110	199	137	249	127	86	155	106	192
600	160	152	280	190	350	190	118	218	146	271
800	220	187	358	234	447	254	146	279	180	345
1000	270	215	426	269	534	318	166	331	206	413
1200	330	244	488	305	610	382	190	378	234	470
1400	380	270	540	338	675	445	210	420	261	521
1500	410	282	564	343	707	477	219	438	272	545
1600	435	292	585	366	732	510	228	455	282	562
1800	490	311	640	388	800	572	241	496	300	610
2000	540	333	671	406	838	636	258	522	320	710

Velocity at Pitch Circle, Feet per Minute	2 Diametral Pitch, 21 Teeth, 10½-inch Pitch Diameter					2½ Diametral Pitch, 21 Teeth, 8.4-inch Pitch Diameter				
	Revolutions per Minute	12½-inch Face		15½-inch Face		Revolutions per Minute	10-inch Face		12½-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	145	64	116	79	144	182	41	74	51	93
600	218	89	164	111	204	273	57	105	71	130
800	291	109	207	136	259	364	70	134	87	167
1000	364	125	250	154	309	455	80	160	100	200
1200	437	143	285	178	355	546	91	185	114	228
1400	510	158	316	196	393	637	101	202	126	253
1500	546	166	329	206	410	683	106	211	132	264
1600	580	171	342	213	425	729	109	220	137	274
1800	655	182	375	226	465	820	116	240	145	300
2000	730	195	392	242	488	910	122	251	152	314

Velocity at Pitch Circle, Feet per Minute	3 Diametral Pitch, 21 Teeth, 7-inch Pitch Diameter					3½ Diametral Pitch, 21 Teeth 6-inch Pitch Diameter				
	Revolutions per Minute	8¼-inch Face		10½-inch Face		Revolutions per Minute	7¼-inch Face		9-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	218	28	51	36	65	255	21	39	26	48
600	327	39	72	50	92	382	29	55	36	68
800	436	48	92	61	117	510	36	70	45	87
1000	546	55	110	70	140	637	42	83	52	103
1200	655	63	126	80	160	765	47	95	58	118
1400	765	69	139	89	177	892	52	105	65	130
1500	820	72	146	93	184	955	55	110	68	136
1600	875	75	151	96	192	1020	57	114	71	141
1800	983	80	165	102	210	1150	61	123	76	154
2000	1090	84	173	107	220	1275	65	131	81	162

and the curves in Fig. 6 were used in calculating the tables, for obtaining the relative toughness of the different metals which are ordinarily used.

The tables give the horsepower transmitted by herringbone gears in which the pinion has 21 teeth, and the width of face corresponds to 8 and 10 times the circular pitch. To find the horsepower for any other number of teeth, ascertain the pitch line velocity, and under the given diametral pitch, find the horsepower corresponding to this velocity. To find the horsepower transmitted by a brass gear, multiply that found for a cast-iron

Horsepower Transmitted by Herringbone Gears

Velocity at Pitch Circle, Feet per Minute	4 Diametral Pitch, 21 Teeth, 5¼-inch Pitch Diameter					5 Diametral Pitch, 21 Teeth, 4.2-inch Pitch Diameter					6 Diametral Pitch, 21 Teeth, 3½-inch Pitch Diameter				
	R.P.M.	6¼-inch Face		7¾-inch Face		R.P.M.	5-inch Face		6¼-inch Face		R.P.M.	4-inch Face		5¼-inch Face	
		C.I.	S.C.	C.I.	S.C.		C.I.	S.C.	C.I.	S.C.		C.I.	S.C.	C.I.	S.C.
400	292	16	29	20	36	364	10	19	13	24	436	7	13	9	17
600	437	22	41	27	51	545	14	27	18	34	655	10	18	13	24
800	584	27	53	34	66	725	17	34	21	43	873	12	23	16	30
1000	730	31	62	39	77	910	20	40	25	50	1090	14	27	18	35
1200	875	36	71	44	88	1090	23	46	29	58	1310	15	31	20	41
1400	1020	40	80	50	99	1270	25	51	31	64	1525	17	35	22	46
1500	1090	41	83	51	103	1360	26	53	32	66	1635	18	36	24	47
1600	1160	43	86	53	107	1450	27	55	34	69	1745	19	37	25	49
1800	1310	46	93	57	116	1630	29	59	36	74	1960	20	40	26	53
2000	1460	49	99	61	122	1820	31	63	39	79	2180	21	42	28	55

gear by 0.8; for gun metal, multiply by 1.25; and for phosphor-bronze, by 1.63. For high-carbon steel forgings, multiply the horsepower transmitted by steel-casting gears by 1.45.

To illustrate the method of using the tables, it is assumed that a herringbone gear drive is required to transmit 100 horsepower from a motor running at 650 revolutions per minute with a speed reduction of 12 to 1. In the table, under $2\frac{1}{2}$ pitch, 10-inch face and opposite 637 revolutions per minute, 101 horsepower is found to be the capacity of a cast-iron gear running at a pitch line velocity of 1400 feet per minute. This would necessitate $\frac{12 \times 21}{2.5} = 100.8$ inches pitch diameter for the gear to mesh with a 21-tooth, $2\frac{1}{2}$-diametral pitch pinion. The pitch line velocity of a 17-tooth pinion would be 1150 (or, say 1200) feet per minute, and in the 1200-foot line, it is seen that a gear running at this speed would transmit 91 horsepower. This means that a wider face gear must be used, if a 17-tooth pinion is to find successful application, the width of face required being $\frac{100 \times 10}{91} = 11$ inches. The pinion in either case should be made of cast steel or preferably of forged steel, to compensate for the greater wear to which it is exposed.

If it is required to find the horsepower transmitted by a 2-pitch cast-steel gear, having a pitch diameter of 90 inches by 10-inch face, running at 50 revolutions per minute, we find that the pitch line velocity is 1180 feet per minute. Opposite 1200 in the table, under 2 diametral pitch and $12\frac{1}{2}$-inch face, we find 285 horsepower, while the capacity of a gear of 10-inch face would be $\frac{10 \times 285}{12.5} = 228$ horsepower. Although the tables are calculated for 21-tooth pinions, they are universal in their application, it being merely necessary to find the corresponding pitch line velocity for any number of teeth.

Summary of Salient Features. — Before describing some special applications of herringbone gears to the needs of various industries and machines, it may be well to summarize their salient features. The smooth and continuous action is virtually independent of the diameter or number of teeth in the pinion. Extremely high ratios of reduction can be used without fear of uneven driving or undue wear and without need for unwieldy gear diameters which would be disproportionate to the general design. High ratio gears of this type transmit power with practically no more loss than low ratio gears. They are far more efficient than belts, ropes, worm-gears or compound trains of spur gearing, while their adoption results in a wholesale reduction of countershafts and bearings, which reduces the power consumption and running costs to a remarkable degree.

There are many instances where spur gears cannot be used because the vibration which they set up has a detrimental effect on the driven machine or its product. The inconvenience of a cumbersome system of belts or ropes has usually to be endured in such cases, but it is not too much to say that the requirements of almost all of these conditions are fully satisfied by this type of herringbone gears.

The application of spur gears has been much restricted by the noise which they make when run at high velocities. The use of rawhide or other soft materials has proved successful for comparatively light work, but is not adapted to low speeds and heavy service. It should be noted that the use of soft pinions, while mitigating the nuisance of excessive noise, does not reduce vibration or unevenness of motion. There is a limit to the pitch line velocities at which spur gears can be operated beyond which it is unsafe to use them. This limit is far below the minimum velocities which can be used in connection with steam turbines of economical design and high power. Accurate herringbone gears operate quite smoothly at velocities which are impossible for other types. This feature would appear to reserve for them a field of application which has great possibilities and is likely to cause some great changes in the standard practice of today.

Application to Steam Turbines.—Direct-connected steam turbines for marine propulsion have been only partially successful in a very limited field. It is only when the power required is very great and the speed of the vessel unusually high that the direct-connected turbine can be applied, and even then the application does not do full justice to either turbine or propeller, while the first cost is much higher than it need be. The use of direct-coupled turbines is confined to fast ocean liners and warships. Ordinary vessels of commerce cannot be adapted to turbine power in this form. Mr. Parsons, of steam turbine fame, attacked the problem of applying the turbine to an ordinary freight steamer of moderate power. To this end he purchased the S. S. Vespasian, a modern tramp, with triple-expansion engines of about 1000 H.P. and a speed of 11 knots, with propeller running at 75 R.P.M. As a preliminary to the installation of geared turbines on this vessel, the original engines were overhauled, and a series of coal consumption trials made under regular sea-going conditions.

The engines were then removed and for them were substituted a pair of steam turbines connected to the propeller by herringbone gears. Each turbine develops about 500 H.P. at 1500 R.P.M. The propeller runs at the original speed of 75 R.P.M. Each turbine is coupled to a herringbone pinion with teeth cut solid on a shaft of soft-grade chrome-nickel steel. The two pinions mesh with rolled-steel gear rings mounted on a cast-iron spider which is keyed to the propeller shaft. The whole gear system is enclosed in a case, and the teeth are kept lubricated by oil jets. The great width of the pinions in proportion to their diameter made it necessary to provide room for bearings between the right- and left-hand teeth. The proportions of this remarkable gear unit are as follows: Pinions, 20 teeth; gear, 398 teeth, 4 diametral pitch; teeth of involute form, 20-degree pressure angle, 23-degree spiral angle; over-all face width, 34 inches, including 10 inches space for bearing; actual face width, 24 inches; ratio of reduction, 19.9 to 1.

When this gear had been running in regular voyages for more than a year and had covered over 20,000 miles, the results that had been obtained proved to have been interesting and satisfactory. The efficiency of the gear was fully 98 per cent, including the losses in the bearings on the gear case. The geared turbine showed a sustained all-around saving in fuel consumption of more than 25 per cent over the original engines. The wear on the teeth was negligible after 20,000 miles, being only 0.002 inch at the pitch lines of the pinions.

Herringbone Gears for Machine Tools. — The field for accurate herringbone gears in connection with machine tool driving is very extensive. For individual motor drives this gear gives a positive transmission which is free from vibration and less noisy than so-called silent chains or rawhide pinions, while there is no trouble from slipping belts or slack chains; but the real advantage of these gears lies in the better finish that can be obtained when they are used for the entire main transmission, and in the higher output combined with reduced maintenance which they give to heavy machine tools. Chatter is eliminated. Even wheels of grinding machines have been successfully driven through herringbone gears. Reversing gears for heavy planers are a revelation to those familiar only with the ordinary spur drive.

Geared Hydraulic Turbines. — The speed of hydraulic turbines is controlled by the available head of water supplied to them. The greater number of turbines are required to operate under low heads and must run at slow speed. Hydroelectric power has usually to be transmitted to a considerable distance and is produced in the form of alternating current of definite periodicity. The speed of the turbines may be as low as 50 revolutions per minute or even less. A large direct-coupled alternator for this speed is an expensive proposition.

Herringbone gears can be used to speed up from the slow-running turbines to generators of normal design, speed and efficiency. The smooth action of these gears is unimpaired when the wheel drives the pinion, and high ratios of speed increase can be obtained from them without noise and with less loss than direct-coupled units will give.

Rolling Mills and Rod Mills. — There are two advantages in the use of accurate herringbone gears for this class of work. The absence of shock in transmission renders breakages much less frequent than with cut spur or molded helical gears. The even transmission and entire elimination of vibration allows the finishing rolls to be gear-driven for the finest work without showing gear marks on the finished product. Herringbone-geared mills run with very little noise. This may be of less consequence in rolling mills than in most other applications, but it is an improvement. Rod mills, with their quantities of high-speed gearing, can be very advantageously equipped with herringbone gears and pinions.

CHAPTER IV

METHODS FOR FORMING THE TEETH OF SPIRAL AND HERRINGBONE GEARS AND WORMS.

Spiral, herringbone and worm gearing are all radically different in their action. The first two forms, however, and the worm member of the third, are identical so far as the principles governing the forming of their teeth are concerned, and they may, therefore, be considered together in this chapter.

Principal Methods Used. — Almost as great a variety of methods of cutting teeth are possible for helical as for spur gears. Commercially, however, the two important principles are the formed tool and the molding-generating methods. The templet, odontographic and describing-generating methods of cutting gear teeth (in each of which the outline is worked out by the *point* of a tool, suitably constrained) are most useful for cutting gears of large size, in which tools acting on the formed tool or molding-generating principle would be subject to too heavy cuts. Since helical gearing is generally confined to small and medium-sized work, these processes are unnecessary, being by nature rather slow in action, and dependent for their accuracy on the preservation of the shape of easily injured points of comparatively small cutting tools. As in the case of spur gears, the molding-generating method is of comparatively recent introduction, and is confined almost wholly to the production of teeth of involute form.

Machines Using Formed Tools in a Shaping or Planing Operation. — With the twisted teeth in gears of the class under discussion, it is evidently necessary, in employing shaping or planing operations, to give a rotary movement to the blank being operated on, at the same time as, and in the proper ratio with, the cutting stroke of the tool. This is necessary to compel the tool to follow the helix on which the teeth of the gear or the worm are to be formed. Attachments have, for example, been made for the shaper, giving the work the proper motion for cutting helical teeth.

In one of these attachments the work is mounted between centers on a supplementary bed, fastened to the work table of the shaper. The faceplate by which the work is driven from the headstock spindle is connected to that spindle by an indexing mechanism, consisting of a notched plate, with a locking bolt for holding the work in the different positions for the different numbers of teeth required. The headstock spindle is connected, by spiral gearing and a set of change gears, with a pinion operated by a rack, which rack is fastened to the shaper ram. It will be seen that this connection with the shaper ram will give a rocking movement to the headstock spindle and the work, in unison with the stroke of the tool. By selecting suitable change gears, this rocking movement may be made of any desired amplitude for a given length of stroke, so that any lead of helix or spiral desired may be obtained. Provision is made, in the mechanism by which the rack is attached to the ram, for raising or lowering the work table to the position required for different diameters of work. The tool is, of course, fed downward by hand, and the indexing is done by hand also.

In another shaper attachment a spur gear keyed to the headstock spindle meshes with a vertical rack, sliding in a guide which is cast integrally with the headstock. This vertical rack is pivoted to a block which slides in a guide attached to a swiveling head, so that the guide may be adjusted to any angle. This swiveling head, in turn, is attached to a bar, which is fastened to the ram, and is guided on ways supported by a framework at the back of the headstock. It will thus be seen that the forward and backward movement of the ram will impart an up-and-down movement to the rack, which will, in turn, give a rocking movement to the spindle of the headstock, and the work which it drives. The amplitude of this rocking can be increased or diminished by setting the swiveling guide at a greater or less angle, so that the helices of various leads can be obtained. This makes the use of change gears unnecessary. The indexing device is similar in principle in the two arrangements.

It will seem strange at first thought, perhaps, to describe the cutting of worms in a lathe as an example of the use of formed tools in shaping or planing operations, but the operation is essentially the same as that involved in shaping spiral gear teeth by means of the attachments described. In Fig. 1 imagine that the lead-screw shown is of such steep pitch that it can be rotated by pushing the carriage backward and forward. Under these circumstances, if provision is made for reciprocating the carriage (corresponding to the ram for the shaper), the lead-screw will be rotated in unison with it, and this movement will be transmitted through change gears *A*, *B*, *C* and *D* to the head-

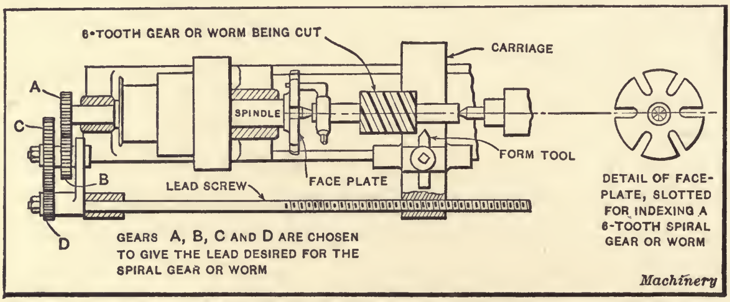


Fig. 1. The Lathe Method of Planing Helical Teeth in Gears or Worms

stock spindle, giving a rocking movement to the work. The only difference in the two cases is that in the lathe a screw of very steep pitch would be used to change the reciprocating motion of the tool to the rocking motion required by the work, while in the case of the shaper the more natural rack and pinion movement is employed.

In the case of the lathe, of course, the power is not applied to the carriage but to the spindle. For that reason it is best adapted for cutting spiral gears of comparatively small lead, or "worms" as they are ordinarily called. If it were attempted to cut 45-degree spirals, for instance, the lead-screw would have to be speeded up so fast, as compared with the movement of the spindle, that the driving belt would be unable to operate the machine. Special lathes have been built for cutting steep worm threads, in which the power has been applied to the lead-screw, the spindle being driven from it through the change gears. A lathe so arranged would have as much difficulty in cutting fine pitches as the ordinary lathe does in cutting coarse ones.

Different methods of indexing may be used for the lathe. It will be noticed that in Fig. 1 the faceplate used has the same number of slots as the required number of teeth. After one tooth space has been cut, the work can be removed, and replaced again between the centers with the tail of the dog in another slot. After this space has been completed, the next one is cut, and so on until the whole six are finished. Other methods are in use, such as slipping of change gears *A* and *B* past each other a certain number of teeth, as determined by calculation.

Special lathes are built for threading, some of which are automatic in their action. One of these is built by the Automatic Machine Co., Bridgeport, Conn. The size especially adapted to cutting worms is provided with mechanism for duplicating the action of a manually-operated lathe engaged in threading. After a piece of work has been placed between the centers and the machine has been started, the work revolves, and the carriage feeds forward until the proper length thread has been cut; then the tool is withdrawn, and the carriage returns to begin again on a new cut — and so on without attention from the operator. The tool is fed in a certain suitable amount, at the beginning of each cut, the amount of this feed being automatically diminished to give a fine finish for the final cuts. When the depth for which the tool has been set is reached, the operation of the mechanism is automatically arrested. In cutting multiple-threaded worms in this machine, multiple tools may be used, thus avoiding the necessity for indexing the work. As many as eight cutting tools have been used at once on this machine, giving a total length of cutting edge of 8 inches.

Machines Using Formed Milling Cutters. — The formed tool or cutter method of shaping the teeth of gears is generally considered as being one in which the tool accurately reproduces its shape in the tooth space it forms. This is true in cutting straight-tooth spur gears, and in planing the teeth of spiral gears by the process just described. It is not exactly true, however, of any possible process of milling spiral teeth. This is best seen in Fig. 2. In the three cases here shown we have first, a planer tool; second, a disk milling cutter; and third, an end milling cutter — all formed to the same identical outline, and cutting helical grooves of the same lead and depth in blanks of the same diameter. The section in each case is a plane one, taken normal to the helix at the pitch line. Of course, the true section to take would be that of the helicoid normal to the helicoid of the groove being cut. The plane in which we have taken the section, how-

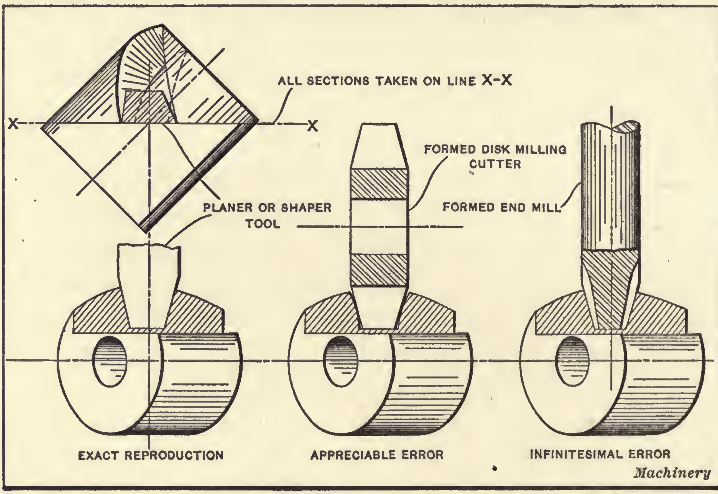


Fig. 2. Comparison of the Accuracy of Form Reproduction obtainable by Formed Planing Tool, Formed Disk Cutter and Formed End-mill

ever, so nearly approximates this helicoid that the error is negligible.

The planer tool necessarily cuts a groove of the same shape as its outline, the plane of its outline being the same as the plane of the section shown. The disk milling cutter, however, interferes with the sides of the groove it cuts. This interference takes place on one side as the teeth are entering, and on the other as the teeth are leaving. This results in a generating action, which takes place in addition to the simple forming action, so that the tooth cut is not an exact duplicate of the outline of the cutter. In the case of the formed end-mill there is also an interference of the same kind as with the formed disk cutter, but it is so slight as to be absolutely undetectable, in all ordinary cases. Its presence is only known from theoretical considerations.

In spite of its imperfect reproduction of the desired form, the disk cutter is the type generally used for milling, since it may be so relieved as to retain its shape even after repeated grinding. The end-mill type of formed cutter cannot remove so much stock in a given time, and it is difficult to make it so that it can be ground without changing its form. The only way in which this grinding can be practically performed is by the use of some form of templet grinding machine. The formed end-mill is nevertheless used to a limited extent.

The simplest way of using the milling process for cutting helical gears or worms makes use of the universal milling machine. With this machine the work and the feed-screw of the table on which it is mounted are so connected by means of gearing that the forward feeding gives a rotary movement to the work, producing a helix of the required lead. The mechanism is identical in principle with that shown in Fig. 1 for the lathe, the only difference being that in the milling process the longitudinal movement is a steady feeding motion, made once for each tooth space, instead of being a continuously reciprocating motion. The simple indexing device shown in Fig. 1 is replaced by the more elaborate index plate and worm-wheel device of the spiral head.

This mechanism is exemplified in the Brown & Sharpe universal milling machine with its spiral head. The work has to be swung at an angle with the cutter to agree with the helix angle at the pitch line. This is done by swiveling the table of the universal milling machine to bring the work to the proper angle with the cutter. In most makes of machines it is inconvenient, if not impossible, to swivel the table to a greater angle than 45 degrees. For greater angles special attachments are provided for swiveling the cutter, leaving the table in its normal position at right angles to the spindle of the machine. These various attachments allow the milling machine to work throughout a wide range of angles for helical gears and worms, the only limitation being one similar to that imposed on worm cutting in the lathe, though the limitation is reversed. For worms or gears of too small lead as compared with their diameter, the rotary movement of the blank is so great that the comparatively slow-moving feed-screw is unable to speed up the spiral head mechanism to get the required movement, and still furnish power enough for feeding the work against the cutter.

Points Relating to the Milling of Spiral Gears. — Before describing other methods for cutting the teeth in spiral or helical gears, we shall briefly cover the essential points to be considered in milling these gears in a universal milling machine. The first point to be considered is the pitch of cutter to be used. The thickness of the cutter at the pitch line for milling spiral gears should equal one-half the normal circular pitch n (see Fig. 3). If a cutter were used having a thickness, at the pitch line, equal to one-half the circular pitch P, as for spur gearing, the spaces between the teeth would be cut too wide, thus producing thin teeth. The normal pitch varies with the angle α of the spiral; hence, the spiral angle must be considered when selecting a cutter. The cutter should be of the same pitch as the *normal* diametral pitch of the gear and this normal pitch is found by dividing the "real" diametral pitch by the cosine of the spiral angle. To illustrate, if the pitch diameter of a spiral gear is 6.718 and there are 38 teeth having a spiral angle of 45 degrees, the "real" diametral pitch equals $38 \div 6.718 = 5.656$; then, the normal diametral pitch equals 5.656 divided by the cosine of 45 degrees or $5.656 \div 0.707 = 8$. A cutter, then, of 8 diametral pitch is the one to use for this particular gear. This same result could also be obtained as follows: If the circular pitch P is 0.5554, the normal circular pitch n can be found by multiplying the circular pitch P by the cosine of the spiral angle. For example, $0.5554 \times 0.707 = 0.3927$. The normal diametral pitch is then found by

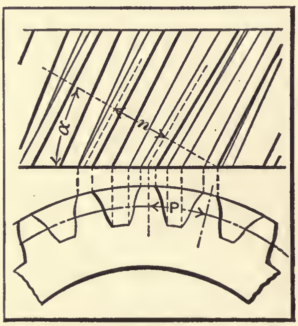


Fig. 3. Relation between Normal and Regular Circular Pitch

dividing 3.1416 by the normal circular pitch. Thus $\frac{3.1416}{0.3927} = 8$, which is the diametral pitch of the cutter.

According to the Brown & Sharpe system of cutters for spur gears having involute teeth, eight different shapes of cutters (marked by numbers) are used for various numbers of teeth in gears of any one pitch. When the diametral pitch is known,

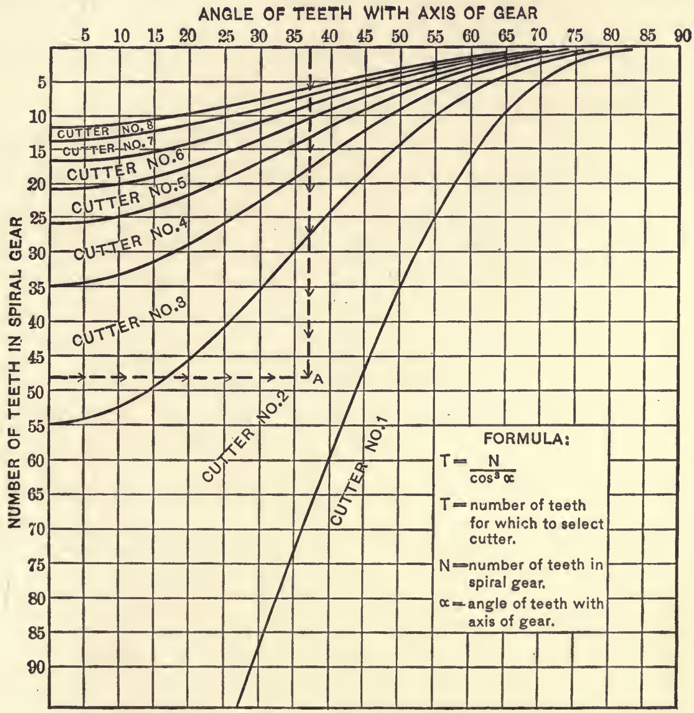


Fig. 4. Diagram for finding Cutter for Milling Spiral Gears

the number of cutter for that particular pitch must, therefore, be determined as explained in Chapters I and II. A diagram and table, useful in this connection, are given in the following.

Diagram for Finding Cutter for Milling Spiral Gears. — A diagram, Fig. 4, has been prepared, giving directly the number of cutter to be used for a given number of teeth and a given spiral angle. The heavy lines drawn in the diagram are division lines between the fields to which each cutter applies. For example, suppose the angle of the teeth of a gear is 37 degrees with its axis, and the number of teeth is 48. The point A, at which the horizontal line representing the number of teeth and the vertical line representing the angle intersect, falls within the area marked cutter No. 2; therefore, a No. 2 cutter is required for cutting a 48-tooth spiral gear having a spiral angle of 37 degrees.

Table for Selecting Cutter for Milling Spiral Gears. — The "Table for Selecting Cutter for Milling Spiral Gears" gives the value of the factor $K = \frac{1}{\cos^3 \alpha}$ which enters in the formula for finding the number of teeth for which to select the cutter for milling spiral gears. The table is used as follows: Multiply the actual number of teeth in the spiral gear to be cut by the factor K, as given in the table opposite the angle of spiral. The product gives the number of teeth for which to select the cutter.

Example. — Angle of spiral = 30 degrees; number of teeth in spiral gear = 18.

Factor K for 30 degrees, as found from the table, equals 1.540. Then, number of teeth for which to select the cutter = 18 × 1.540 = 28, approximately. Hence, use spur gear cutter for 28 teeth, or cutter No. 4.

Angular Position of Table when Milling Spiral Gears. — In cutting a spiral gear in a milling machine as ordinarily arranged, it is necessary to set the table to the helix angle in order that the sides of the cutter may not interfere, or drag in the cut; but the helix angle varies with the depth, being greatest at the top of the tooth, less at the pitch line and still less at the bottom of the cut. In fact, if the cut were deep enough to reach all the way to the center of the piece being operated on, the helix angle would become zero, or parallel to the center line. If mechanics in general were asked what would be the proper angle at which to set the table, they would, in most cases, say that the helix angle at the pitch line would be the one to determine the setting. This setting has the effect of making the width of the cut exactly right at the pitch line, but it does so at the expense of undercutting and weakening the teeth.

It has, therefore, been frequently stated that the most suit-

Table for Selecting Cutter for Milling Spiral Gears

Angle of Spiral, α	K	Angle of Spiral, α	K	Angle of Spiral, α	K	Angle of Spiral, α	K
0° 0′	1.000	21° 0′	1.228	42° 0′	2.436	63° 0′	10.69
0° 30′	1.000	21° 30′	1.241	42° 30′	2.495	63° 30′	11.27
1° 0′	1.001	22° 0′	1.254	43° 0′	2.557	64° 0′	11.87
1° 30′	1.001	22° 30′	1.268	43° 30′	2.621	64° 30′	12.55
2° 0′	1.002	23° 0′	1.282	44° 0′	2.687	65° 0′	13.25
2° 30′	1.003	23° 30′	1.297	44° 30′	2.756	65° 30′	14.03
3° 0′	1.004	24° 0′	1.312	45° 0′	2.828	66° 0′	14.86
3° 30′	1.005	24° 30′	1.328	45° 30′	2.902	66° 30′	15.80
4° 0′	1.007	25° 0′	1.344	46° 0′	2.983	67° 0′	16.76
4° 30′	1.009	25° 30′	1.360	46° 30′	3.066	67° 30′	17.85
5° 0′	1.011	26° 0′	1.377	47° 0′	3.152	68° 0′	18.98
5° 30′	1.013	26° 30′	1.395	47° 30′	3.242	68° 30′	20.33
6° 0′	1.016	27° 0′	1.414	48° 0′	3.336	69° 0′	21.72
6° 30′	1.019	27° 30′	1.434	48° 30′	3.436	69° 30′	23.33
7° 0′	1.022	28° 0′	1.454	49° 0′	3.540	70° 0′	25.00
7° 30′	1.026	28° 30′	1.474	49° 30′	3.650	70° 30′	26.97
8° 0′	1.030	29° 0′	1.495	50° 0′	3.767	71° 0′	28.97
8° 30′	1.034	29° 30′	1.517	50° 30′	3.887	71° 30′	31.40
9° 0′	1.038	30° 0′	1.540	51° 0′	4.012	72° 0′	33.88
9° 30′	1.042	30° 30′	1.563	51° 30′	4.144	72° 30′	36.92
10° 0′	1.047	31° 0′	1.588	52° 0′	4.284	73° 0′	40.00
10° 30′	1.052	31° 30′	1.613	52° 30′	4.433	73° 30′	43.88
11° 0′	1.057	32° 0′	1.640	53° 0′	4.586	74° 0′	47.79
11° 30′	1.062	32° 30′	1.667	53° 30′	4.752	74° 30′	54.72
12° 0′	1.068	33° 0′	1.695	54° 0′	4.925	75° 0′	57.68
12° 30′	1.074	33° 30′	1.724	54° 30′	5.101	75° 30′	64.15
13° 0′	1.080	34° 0′	1.755	55° 0′	5.295	76° 0′	70.65
13° 30′	1.087	34° 30′	1.787	55° 30′	5.497	76° 30′	79.20
14° 0′	1.094	35° 0′	1.819	56° 0′	5.710	77° 0′	87.78
14° 30′	1.102	35° 30′	1.853	56° 30′	5.940	77° 30′	99.50
15° 0′	1.110	36° 0′	1.889	57° 0′	6.190	78° 0′	111.3
15° 30′	1.118	36° 30′	1.926	57° 30′	6.435	79° 0′	144.0
16° 0′	1.127	37° 0′	1.963	58° 0′	6.720	80° 0′	191.2
16° 30′	1.136	37° 30′	2.003	58° 30′	7.010	81° 0′	261.4
17° 0′	1.145	38° 0′	2.044	59° 0′	7.321	82° 0′	370.6
17° 30′	1.154	38° 30′	2.086	59° 30′	7.650	83° 0′	552.1
18° 0′	1.163	39° 0′	2.130	60° 0′	8.000	84° 0′	876.4
18° 30′	1.172	39° 30′	2.176	60° 30′	8.380	85° 0′	1509.0
19° 0′	1.182	40° 0′	2.225	61° 0′	8.780	86° 0′	2940.0
19° 30′	1.193	40° 30′	2.275	61° 30′	9.209	87° 0′	6990.0
20° 0′	1.204	41° 0′	2.326	62° 0′	9.658	....	.....
20° 30′	1.216	41° 30′	2.380	62° 30′	10.160	....	.....

able angle (and the one most likely to produce the best results) at which to set the table of the milling machine when milling spiral gears is that corresponding either to the diameter of the gear measured at the bottom of the space, or to the diameter measured at the working depth. The reason invariably adduced for this is, as just mentioned, that, if the angle chosen is the angle of the spiral measured on the pitch cylinder of the gear, an undue amount of undercutting, and therefore weakening, of the teeth will occur, owing to an excessive amount of interference with the sides of the teeth on the part of the cutter;

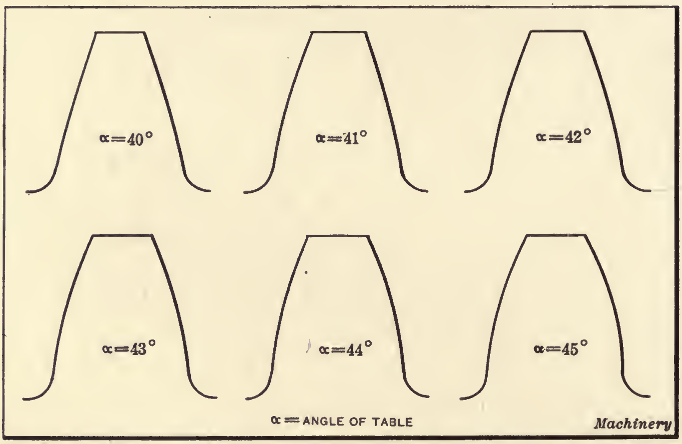


Fig. 5. Shapes of Teeth Obtained by Setting the Table at Different Angles, the Cutter and the Lead remaining the Same

and that, therefore, a somewhat smaller angle should be selected to reduce these effects.

To determine whether there was, practically, anything in this idea or not, some experiments were recently made on a spiral gear, the immediate object of the experiments being to find out what the effect of altering the angle of setting of the milling machine table was upon the shape of the tooth cut.

The experiments were made upon a cast-iron gear, with a pitch diameter of 4.242 inches, and designed for 24 teeth, the diametral pitch (corresponding to the normal circular pitch) being 8. The correct cutter to use was determined by the formula $N_e = \frac{N}{\cos^3 \alpha}$, this cutter being No. 3 in each of the cases dealt with. The experiments consisted of cutting six teeth in the gear blank, all being of the same depth, the angle of setting of the table of the milling machine being different in each of the six cases. The spiral angle measured on the pitch cylinder was 45 degrees, the lead of the spiral being 13.32 inches, for which the gears of the spiral dividing-head were arranged. The six spirals chosen were at angles of 45, 44, 43, 42, 41 and 40 degrees, each tooth being formed by two cuts at one angle, the lead of the spiral remaining the same throughout the series of tests. It should be here noted that 43 degrees is the angle which corresponds to the diameter measured at the bottom of the space.

The profiles of the teeth taken as sections normal to the spiral on the pitch surface are indicated in Fig. 5, the profiles being

Table of Observed Tooth Dimensions

Angle of Table Setting, Degrees	Depth at which Tooth is Measured						
	0	0.050	0.100	0.125	0.150	0.200	0.250
	Width of Tooth						
45	0.104	0.145	0.185	0.200	0.203	0.221	0.239
44	0.102	0.144	0.181	0.195	0.202	0.220	0.238
43	0.099	0.142	0.176	0.188	0.200	0.218	0.236
42	0.094	0.135	0.168	0.180	0.196	0.215	0.234
41	0.087	0.128	0.158	0.171	0.185	0.211	0.232
40	0.078	0.115	0.146	0.158	0.171	0.205	0.230

drawn accurately to scale — three times full size. The various widths of the teeth at different depths were obtained as accurately as possible by means of a Brown & Sharpe gear-tooth vernier caliper. These widths are given in the accompanying table. Of course, it will be readily seen that although great care was exercised in securing measurements that would be as accurate as possible, the dimensions given above may be incorrect by about one or two thousandths inch, but not more.

In regard to the shapes of the teeth, it will be noticed that the 45-degree tooth is slightly undercut at the root, while the other teeth do not show any undercutting whatever. The undercutting referred to in the 45-degree tooth amounts to a reduction in width below the widest part of the tooth of about 0.010 inch.

The deductions drawn from the results of these tests are:

1. That the practice of setting the table at an angle less than the spiral or helix angle measured on the pitch surface is justified; although this angle should not be less than the spiral or helix angle measured at the bottom of the tooth.

2. That a cutter for a larger number of teeth than that given by the formula $N_e = \frac{N}{\cos^3 \alpha}$ could probably be employed, in order to counteract the flattening and widening effect of the cutter with an angle as indicated above.

In spite of the good reasons given for setting the table to the angle determined by the root of the teeth, it is common practice to set the table to the spiral angle of the teeth at the pitch line. In any case, the angle is determined by first obtaining the tangent of the angle, and then finding the corresponding angle from a table of tangents. For example, if the pitch diameter of the gear is 4.46 and the lead of the spiral, 20 inches, the tangent equals $\frac{4.46 \times 3.1416}{20} = 0.700$, which is the tangent of 35 degrees; therefore the table should be swiveled 35 degrees from its position at right angles to the spindle.

Milling the Spiral Teeth. — After a tooth space has been milled, the cutter should be prevented from dragging through it when being returned for another cut. This can be done by lowering the blank slightly, or by stopping the machine and turning the cutter to such a position that the teeth will not touch the work. If the gear has teeth coarser than 10 or 12 diametral pitch, it is well to take a roughing and a finishing cut. When pressing a spiral gear blank on the arbor, it should be remembered that it is more likely to slip when being milled than a spur gear, because the pressure of the cut, being at an angle, tends to rotate the blank on the arbor.

Specialized Forms of Milling Machines for Cutting Spirals by the Formed Cutter Method. — The principle of the universal milling machine for cutting spiral gears and worms has been applied to the design of various special machines for the same purpose. The specialization of these machines includes making the spiral and indexing mechanisms integral parts of the tool, so that they have a much greater capacity for taking heavy cuts than is the case where they are merely attachments.

In Fig. 6 is shown a diagram of the index worm connections of a universal gear cutting machine made by J. E. Reinecker, of Chemnitz-Gablenz, Germany, as arranged for cutting helical gears by the formed milling process. The machine is arranged on the general lines of a milling machine, except that the work spindle is at the top of the column, and the cutter spindle on the knee.

The cutter is driven by an internal gear of large diameter and is mounted on a swivel table which can be set to the required helix angle. The form of cutter slide will provide for any angle up to 30 degrees. For greater angles it is replaced with a slide which can be rotated to any angle throughout the whole circle.

The screw which feeds the cutter slide along the knee is driven from cone pulley *D*, through a vertical shaft and gear connections. Cone pulley *D* is also connected with change gearing *F*, which is, in turn, connected with the index worm, so as to rotate index wheel *G* and the work, for any desired helix. The principle of this is the same as in the universal milling machine, change gears *F* acting the same as the change gears used to connect the spiral head with the table feed-screw. Now the worm-wheel *G* is used for indexing, as well as for rotating the work for the helix, in unison with the feeding of the cutter slide. The way in which these two motions are imparted to *G* without interfering with each other, may be understood from the following description.

At *H* are mounted the change gears by which the indexing is accomplished. These gears drive bevel gear *J*. Index worm *K*, meshing with index worm-wheel *G*, is mounted on a hollow sleeve, keyed fast to the bevel gear *L*. Shaft *M* carries a hub with projecting pivots on its right-hand end, on which are mounted bevel pinions *N*. Shaft *M* is driven by worm-wheel *O*, connected with the feed of the slide cutter through change gears *F*. Gears *J*, *L* and *N* form a differential mechanism of the well-

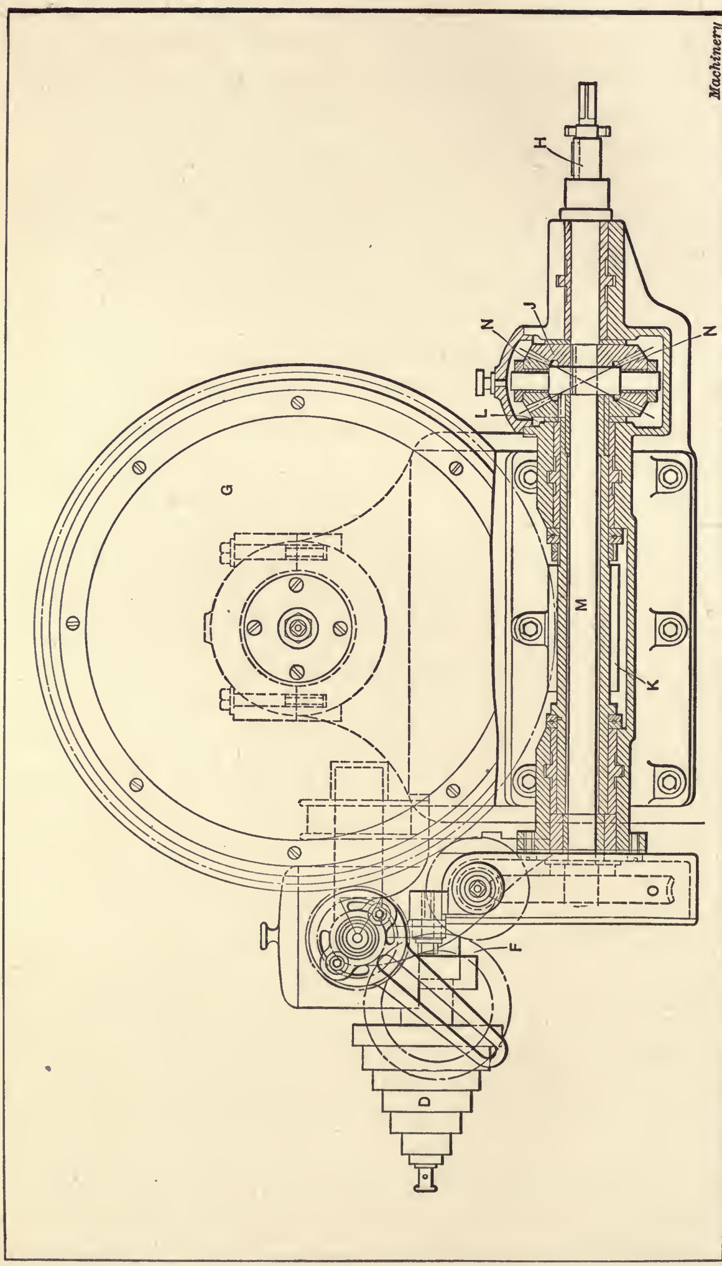


Fig. 6. Section through Reinecker Universal Gear Cutting Machine showing the Differential Mechanism

known "jack-in-the-box" type. The action of this mechanism is such that if shaft *M* be at rest, change gears at *H* may be operated for the indexing, transmitting the motion from gears *J* to *L* through pinions *N* as idlers, thus revolving index worm *K*. On the other hand, with the indexing mechanism still and the cutter slide feeding, the movement thus imparted to shaft *M* may be transmitted (by the rolling of pinions *N* on stationary bevel gear *J*, and the consequent rotation of bevel gear *L*), to worm *K*, and thence to worm-wheel *G* and the work. It will thus be seen that the indexing, and the rotation for the helical cutting, can take place independently of each other. But more than this, the two motions can be operated together without interference. In fact, either of the motions imparted to shaft *M* or gears at *H* may be stopped or reversed independently, and each will have its proper influence on the index wheel and the work.

With this understanding of the differential mechanism, the operation of the machine is easily comprehended. Change gears *H* are connected through a one-revolution friction trip with the main driving shaft. The cutter, set at the proper angle, is fed forward through the work, which is rotated by change gears *F*, shaft *M* and worm *K*, at the proper rate to cut the proper helix. The cutter is then dropped down to clear the work (provision for this being made in the machine), and returned, ready to begin on a new tooth. The indexing mechanism is then tripped by hand, and the work is rotated into position for the new tooth by change gears at *H*, gear *J* and worm-wheel *K*. This is repeated until the gear is done.

In the machine described power is applied to the feed-screw, from which the work is rotated through change gearing. This arrangement is best for helices of great lead. When it comes to milling helical gears with small leads, or worms, it is necessary to use the lathe principle and apply the power to rotating the work, the longitudinal feed being driven from the work spindle through change gearing.

Specialized Form Milling Machines for Herringbone Gears. — A machine for helical gear cutting, but provided with some special features, is used by C. E. Wuest & Co., Seebach, Zurich, Switzerland, for cutting herringbone gears of a special form, in which it is unnecessary to cut the two halves separately, in separate sections, as is the usual case. (See preceding chapter.) The cuts are staggered so that the teeth on one side run into the spaces on the other, in such a way as to permit cutting them with rotary cutters without having one side interfere with the other. The machine for doing this is built on a very simple plan. It consists of a vertical spindle carrying the work, which is indexed by power. The indexing wheel is connected by the usual change gearing with the two vertical slides on which the cutters are mounted on either side. These cutters work simultaneously, one feeding downward to cut the upper half, while the other is feeding upward to cut the lower half.

There is another specialized form of herringbone gear made by André Citroen & Co., Paris, France. The teeth of these gears are shaped by an end milling cutter, guided by suitable mechanism to produce the continuous "wavy" form of herringbone teeth characteristic of these gears (see Fig. 1, Chapter III). This process also has the advantage of not requiring the blank to be made in two pieces. The same principle has been applied by the builders to the cutting of herringbone bevel gears.

Other manufacturers make use of the formed end-mill to a limited extent. The arrangement devised by Gould & Eberhardt for milling large helical gears in the lathe used this form of cutter, and the worms or spiral gears which drive the racks of the Sellers drive planers, made by at least one of our prominent planer builders, are cut by end-mills in a specialized milling machine of simple design, made especially for this purpose.

Automatic Machines for Milling with Formed Cutters. — A number of full automatic machines have been built in an experimental way for milling spiral gears with formed cutters. They have usually been modelled after the automatic spur gear cutter. Evidently the mechanism has to be considerably more complicated. The first complication involved is due to the fact that the index wheel must be under the influence of both the helical and the indexing movements, as in the Reinecker machine in Fig. 6. The differential gearing there shown is the arrangement generally used to effect the combination of these movements in the automatic helical gear cutter.

Another complication is introduced by the necessity for relieving the cutter on its return stroke, after finishing the forward feed through the blank. Backlash in the rotating mechanism between the cutter slide and the work so alters the position of the cutter and the work, on the return stroke, that the latter will drag on the one side of the groove it has just cut, unless it

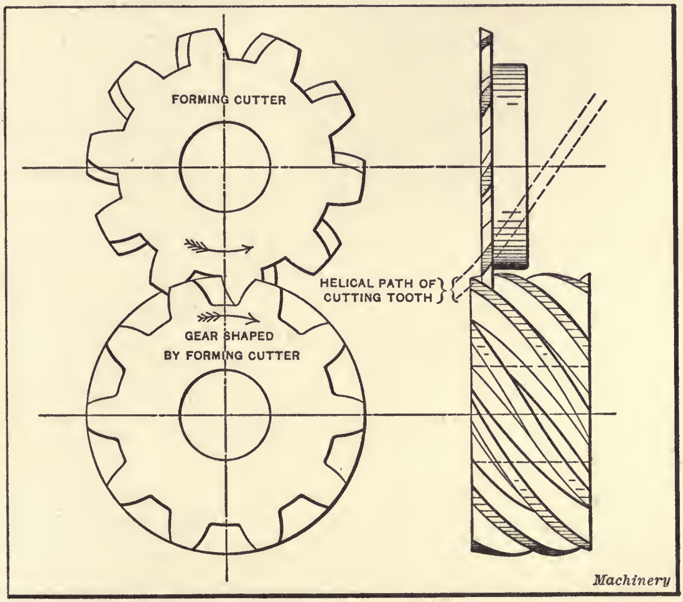


Fig. 7. The Molding-generating Principle arranged to Employ a Cutter having a Helical Shaping Action Cutting Teeth in a Solid Blank

is separated slightly from it. This has been done in various ways in the various machines built; in some cases by mounting the cutter on a supplementary holder which rocks back out of the way on the return stroke, and in other cases by withdrawing the work by mechanism provided for the purpose.

These various complications seem to have militated against the commercial success of the automatic spiral gear cutting machine to such an extent that, so far as the author knows, but one of the various designs built has ever left the shop where it was made.

Molding-generating Principle: Planing Operations. — Passing by the templet, odontographic and describing-generating principles, for the reasons mentioned in the introduction to this chapter, we come to the molding-generating principle. This is applied to helical gears in the same way as to spur gears, with such modifications as are necessary to allow for the helical shape of the teeth. In Fig. 7 the forming cutter and the blank to be cut are rolled together, while the forming cutter is reciprocated

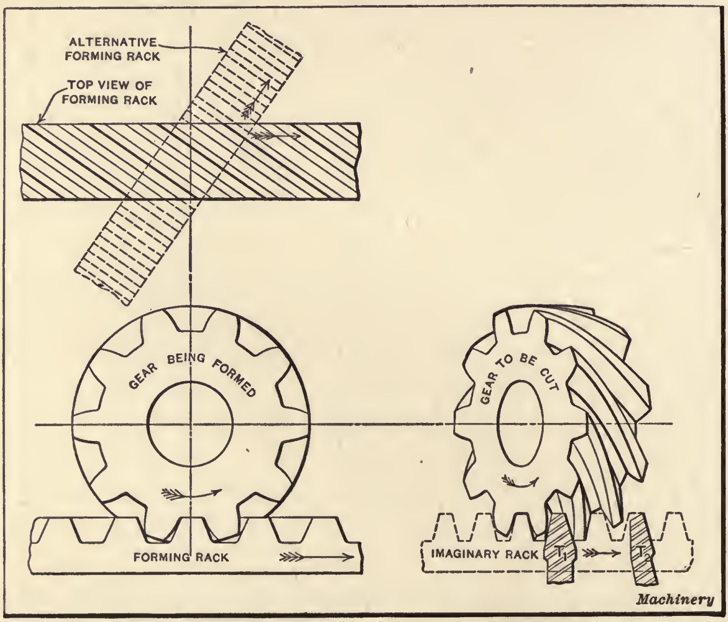


Fig. 8. A Rack with Teeth set on an Angle Operating by Impression on the Molding-generating Principle to Form Teeth in a Helical Gear

Fig. 9. Shaper Tools Representing Teeth of an Imaginary Rack Operating on the Molding-generating Principle in a Helical Gear

axially. In combination with the axial movement, however, the cutter has to be given a rocking movement about its center line, so that its teeth will follow the path of the dotted lines shown, which indicate the helix of the spiral gear which the cutter represents.

In Fig. 8 the forming rack has teeth set on the same angle as the helix angle desired in the gear being formed. The rolling of a plastic blank over this forming rack will form in the blank helical teeth of the shape desired. A top view of the rack is shown, which will make this clearer. Instead of the forming rack shown by the full lines, one like that shown in the dotted lines may be used, whose teeth coincide with those of the first, but which moves in a direction at right angles to the direction of its teeth. If this dotted rack is moved at such a rate of speed that its teeth always coincide with those of the rack shown in full lines, they will evidently both form teeth of exactly the same shape in the blank.

In Fig. 9 we have the dotted rack of the top view of Fig. 8, shown engaged in the operation of generating the teeth of a gear identical with that in Fig. 8. This view has been taken at an angle so as to show the normal view of the rack. If the proper relative rates of rotation of the work and movement of the rack are maintained in Figs. 8 and 9, and the normal sections of the racks in each case are the same, the gears generated will be the same. It is evident in Fig. 9 that the teeth of the rack may be replaced by shaper or planer tools T_1 and T_2, which may be used in forming teeth on the blank by rotating the gear and moving the tools endwise, in the proper ratio prescribed by the conditions in Fig. 8.

Fig. 9 is interesting in that it hints at the principle on which the action of the helical gearing is based. As drawn, it shows very plainly the action of the well-known Sellers drive for planers. It will be noted that for a short space the rack teeth exactly fill the outline of the gear tooth. Contact between the gear and the rack takes place on straight lines running diagonally across the plane faces of the rack teeth.

Practical application has been made of the principle shown in Fig. 9. The Bilgram spiral gear planing machine involves this principle. The work is mounted on a spindle carried in a head, which swivels about a vertical axis so that it may be set to the helix angle of the gear being cut. The cutting tool, having a shape to represent a tooth of the imaginary generating rack, is carried by a ram which works in and out, cutting on the return stroke. This ram is carried by a head which is fed along the bed of the machine. This longitudinal feeding of the ram-carrying head is connected with the rotary movement of the work spindle by change gearing, in the proper ratio for the case in hand, so that the gear will roll with the movement of the head just as it would if it were acting under the influence of the imaginary rack, one of whose teeth is represented by the cutting tool. The conditions are thus exactly the same as in Fig. 9.

Under these conditions, if the machine is set properly, the cutting tool will start to work at one side of the blank, and pass through it, feeding at the end of each successive stroke, with the work rolling in such a way as to form a tooth space of the proper shape. This action is modified somewhat by the method of indexing adopted. The arrangement used indexes the work at every stroke, so that when the tool has once passed through the work, the gear is entirely completed, every tooth having been worked on. This indexing movement and the rolling motion required for the generating are superimposed on each other by suitable mechanism so that neither interferes with the other.

It may be mentioned incidentally that this machine is the only one known to the author in which *all* the requirements for theoretical accuracy in cutting helical gears have been taken care of. There is a minute, although actual, error involved in even the otherwise perfect hobbing process for cutting these gears.

The Hobbing Modification of the Molding-generating Principle. — Instead of using the shaper or planer tool to take the place of the teeth of the imaginary rack shown in Fig. 9, a hob may be used in the same way as for hobbing spur gears. This condition is shown in Fig. 10, which should be compared with Fig. 9. The upper or plan view best shows the respective angular settings of the work and the hob. The hob is set at an angle with the line of movement of the imaginary rack equal to its own helix angle, as for spur gears. The gear being cut is set at an angle with this same line equal to its own helix angle, so that in this case (in which both gear and hob are right-hand) they are set at an angle to each other equal to the difference between the helix angles. If the hob represented by the worm in the diagram is revolved in the direction shown, its teeth will have the same outline and the same movement as the teeth of an imaginary rack, moving in the direction shown. If the work be revolved in the proper ratio with the hob, the latter will form the teeth in the former in the same way that the imaginary rack would, provided it is fed progressively through the work in the direction of line *XX*.

This necessity for feeding the hob through the work introduces an added complexity to the machine in the case of spiral gears, beyond that needed for the spur gear hobbing machine. To understand this, suppose that in Fig. 10 the spindle mechanism

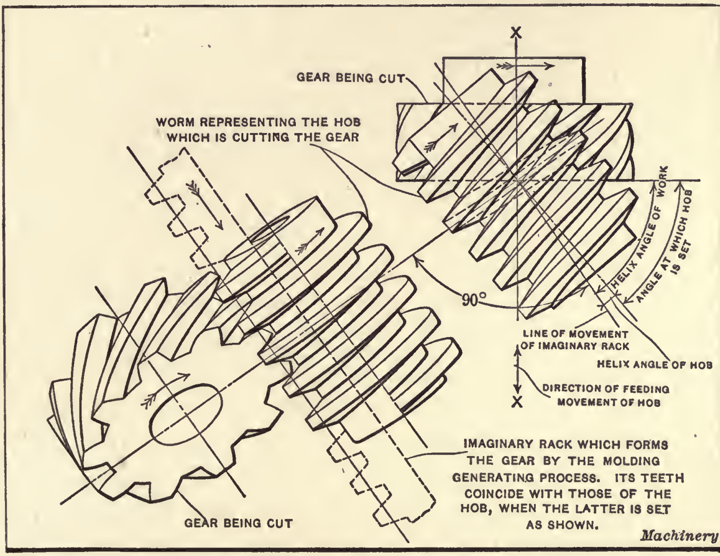


Fig. 10. Molding-generating Method for Cutting Spiral Gears as Exemplified in the Hobbing Process

is stopped, so that both the spindle and work cease to revolve. To make it possible to feed the hob through the work in the direction of line *XX* without having the teeth of the one strike against the other, it will be necessary to revolve either the work or the hob. Suppose that the work be connected by change gearing with the feed-screw of the cutter slide, so that it is revolved as the cutter is fed up or down. Under these conditions the cutter may be moved through the work freely, the latter revolving to allow the cutter to pass. Not only must the work revolve in a definite relation with the feeding of the cutter slide, but the work must also revolve in unison with the cutter or hob, as for spur gears. It must then be so connected with the cutter and with the cutter-slide feed-screw that it will be under the influence of either or both of them, without any interference of the two movements with each other. This connection is usually made by a "jack-in-the-box" or differential mechanism, exactly identical in principle with that shown in Fig. 6 for combining the indexing and helical feeding movements for revolving the work in the Reinecker universal machine. In the case of the

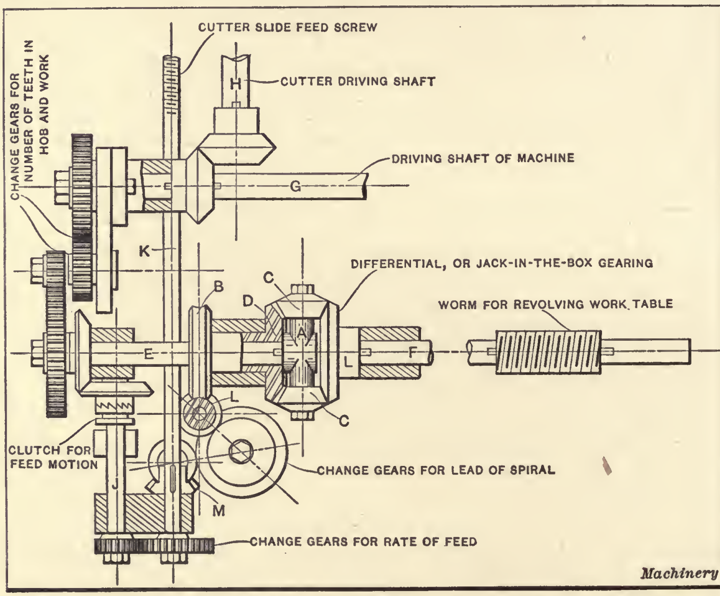


Fig. 11. Typical Arrangement of Gearing for Spiral Gear Hobbing Machine

spiral gear hobbing machine we have a helical feeding movement and a cutter spindle movement to combine for revolving the work.

A typical arrangement of the mechanism used for this purpose is shown in diagrammatic form in Fig. 11. Power is applied to the machine through driving shaft *G*. The bevel gears shown connect this driving shaft with vertical shaft *H*, by means of which the hob is driven. Change gears shown connect shaft *G* with shaft *E*. Considering for the time being that worm-wheel *B* and the attached bevel gear *D* are stationary, the rotation of *E* and the cross-arm *A* keyed to it will cause bevel gears *C* to roll around on stationary gear *D*, thereby revolving gear *L* and shaft *F* to which it is keyed, thus rotating the work table. The change gears connecting *G* and *E* are selected to give the proper ratio of movement between the hob or cutter spindle, and the work table, to agree with the number of threads in the hob and the number of teeth in the gear being cut. The cutter-slide feed-screw *K* is connected by change gears with shaft *J*, which is, in turn, connected through the clutch and the bevel gears shown with shaft *E*. The clutch furnishes the means of stopping and starting the feed, and the change gears serve to give the rate of feed desired. Change gears are also provided connecting bevel gear *M* on feed-screw *K*, with worm *L*, which drives worm-wheel *B*, running loosely on shaft *E*. By this means, supposing for the moment that shaft *E* and its attached cross-arm *A* are stationary, the rotation of the feed-screw is communicated through the change gears to worm-wheel *B* and its attached bevel gear *D*, which, driving bevel pinions *C* on their stationary studs, revolve gear *L*, and with it shaft *F* and the worm driving the work table. In this way, by selecting suitable change gears, the work may be revolved to agree with the length of the lead of the spiral on which its teeth are formed, so that the cutter may be fed up and down through it without interfering with the teeth.

It will be seen that the mechanism shown in Fig. 11 may be arranged to connect the hob and the work in the proper ratio, as for cutting spur gears, and also for connecting the feed-screw and the work in the proper ratio as for cutting spiral gears in the milling machine. This mechanism not only performs these two functions separately, but it will perform them together, as well, so that either the feed or the cutter revolving mechanism may be started, stopped or reversed independently of the other movement, and the work will still be properly controlled under all conditions. The mechanism shown is not that invariably used, but it is typical of the arrangement employed in many hobbing machines designed for cutting helical gearing.

Field of the Hobbing Process for Helical Gears. — There are some limitations to the hobbing process for cutting helical gears. It is not particularly successful in the cutting of gears of such small lead and great helix angle that they would be classed as worms, rather than spiral gears. For such cases the rate of rotation which has to be given the blank is so great in proportion to the downward feed of the cutter by which the rotation is effected (through the change and differential gearing) that it is almost impossible to drive it, the difficulty being the same in kind, though reversed in direction, as that met with in cutting very steep pitches in the lathe. By a slight complication of the machine, however, mechanism could be introduced to overcome this difficulty, and make the hobbing machine universal for all kinds of gears within its range.

In discussing the hobbing processes for cutting spur gears, it is often stated that its field is not yet definitely determined. It may be said, on the whole, that there is no such indefiniteness in regard to the field of the hobbing machine for cutting helical gears. With a well-constructed machine and with hobs of proper shape, spiral gears can be cut more accurately and cheaply by this method than by any other process known. There are none of the mechanical difficulties of indexing and relieving to be taken care of as is the case in automatic machines working on the formed cutter process; and there are none of the uncertainties as to tooth shape due to interference met with in cutting a helical groove with a formed cutter, as shown in Fig. 2. There has been some little difficulty in getting the correct shape of teeth by the hobbing process, due to the elasticity of the mechanism connecting the hob and the work, and to errors in the construction of the hob itself. These difficulties, however, will surely disappear with further experience and investigation.

Apparently the recent rapid development of the hobbing process for cutting spiral gears is the solution of a problem which has long seemed somewhat perplexing. The flexibility of the spiral gear, and the numerous advantages of the herringbone or the twisted-tooth spur gear for transmitting great power noiselessly and smoothly even at high velocities, have long been appreciated, but their extended use has waited for the development of some accurate and inexpensive method of forming helical teeth.

Calculating Gears for Generating Spirals on Hobbing Machines. — From time to time formulas have been developed for calculating the gears to be used for generating spiral gears. Those published in the past, however, have applied only to certain types of gear-hobbing machines. In the following a formula is given which was first published in MACHINERY, December, 1911, and which is applicable to any type of gear-hobbing machine, and which is simpler to use than any formula that had been published up to that time. In developing this formula, simple arithmetical expressions have been made use of, as far as possible, in order to make it especially useful to the practical man.

In order to clearly understand the use of any formula, it is necessary to know something of the principles involved. Fig. 12 shows a top view of a standard hobbing machine (the No. 3 Farwell) designed for cutting spur gears. Before dealing with the change-gear ratios for spiral work, it will be well to have the methods for cutting spur gears properly understood. Assume the hob to be single threaded. It is evident that for each revolution of the hob, the gear being cut must move one tooth. Therefore, the hob revolves, for each revolution of the blank, as many times as there are teeth to be cut. To cut 44 teeth, the table must be geared to revolve once for every 44 revolutions of the hob.

The bevel gearing at D, Fig. 12, has a ratio of 3 to 1, the worm at E is double-threaded, and the worm-wheel F has 40 teeth. Hence, the shaft B must revolve 3×44 times for each revolution of the table, and the worm shaft C must revolve 20 times for each revolution of the table. Hence, we have:

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } C} = \frac{3 \times 44}{20}$$

Inverting this ratio to get the change-gear ratio required to obtain this result, we have:

$$\frac{20}{3 \times 44} = \frac{\text{Product of No. of teeth in driving gears}}{\text{Product of No. of teeth in driven gears}}$$

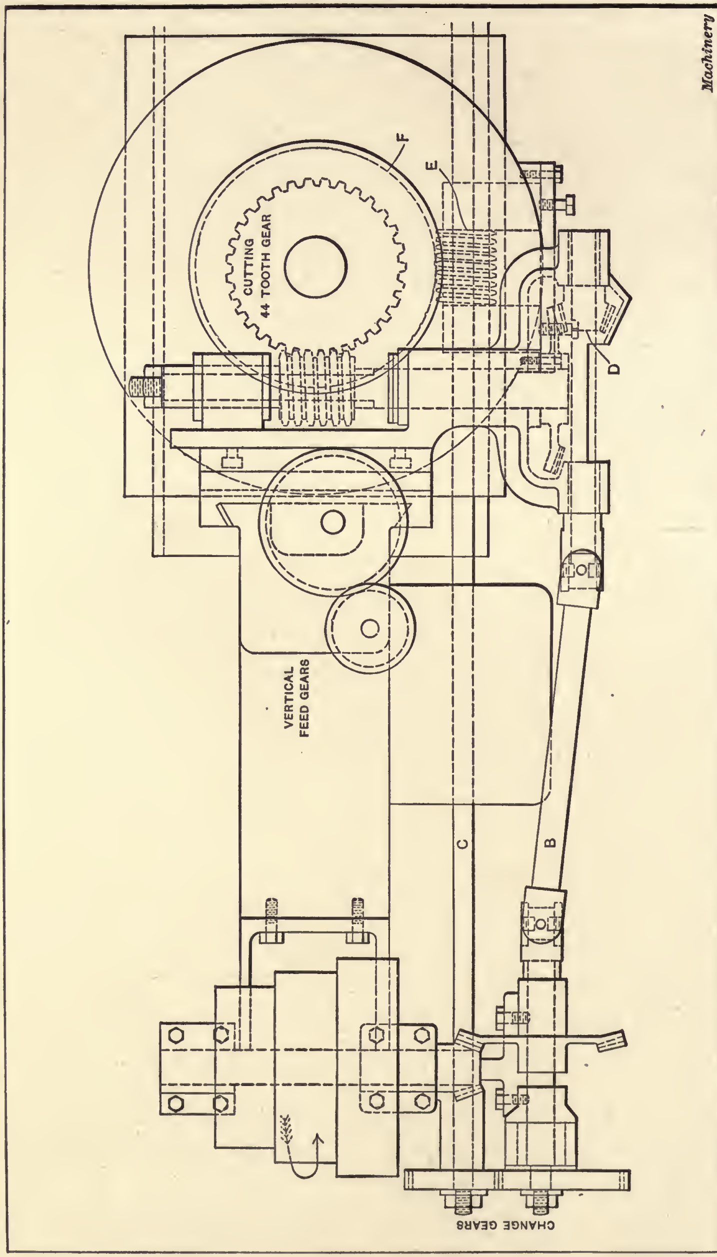


Fig. 12. Hob and Table Driving Arrangement of a No. 3 Farwell Gear Hobbing Machine

In the following formulas we will designate the product of the number of teeth in the driving gears P, and the product of the number of teeth in the driven gears p.

Should we use a double-threaded or triple-threaded hob, the gear we are cutting must revolve two or three teeth for each revolution of the hob; in other words, the speed of the table is increased directly as the number of threads on the hob, so we must multiply the number of teeth in the driving gears by the number of threads on the hob, giving us this formula:

$$\frac{20 \times \text{No. of threads on hob}}{3 \times \text{No. of teeth to be cut}} = \frac{P}{p}$$

A similar formula may be worked out in this way for any type of gear hobber.

Generating Spirals. — For each revolution of the table the head carrying the hob feeds down a certain distance across the face of the blank, this distance varying from 0.010 to 0.150 inch in common practice. To fully understand the following discussion, the action of the machine, as illustrated in Figs. 13 to 16, inclusive, should be noted. In Fig. 13 is shown the generation of a right-hand spiral gear with a right-hand hob; in Fig. 14 a left-hand spiral gear with a right-hand hob; in Fig. 15 a left-hand spiral gear with a left-hand hob; and in Fig. 16 a right-hand spiral gear with a left-hand hob. In each of these illustrations the direction of rotation of the table is indicated by the arrow showing the direction of rotation of the gear being cut. The direction of rotation of the hob is also indicated by an arrow showing the direction of rotation of its shaft. In Figs. 13 and 15, where a gear is cut with a hob of the same "hand," the angle α, as indicated, equals the difference between the tooth angle and the thread angle of the hob. In Figs. 14 and 16, where the gear and the hob are of different "hand," the angle α equals the sum of the tooth angle and the thread angle of the hob. After this preliminary introduction, we are ready to deal intelligently with the problem in hand.

Assume the spiral gear shown in Fig. 17 to have sixty-four teeth. As indicated, the gear has a left-hand spiral and we will assume that it is cut with a left-hand hob. A single-threaded hob cutting a spur gear would revolve sixty-four times for one revolution of the table; but since in this case the teeth are helical and the hob travels downward a certain distance, the position of the gear tooth must be advanced somewhat for every revolution with relation to the hob. In other words,

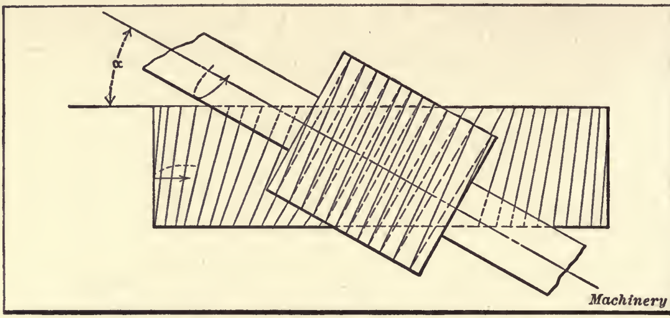


Fig. 13. Cutting a Right-hand Spiral Gear with a Right-hand Hob

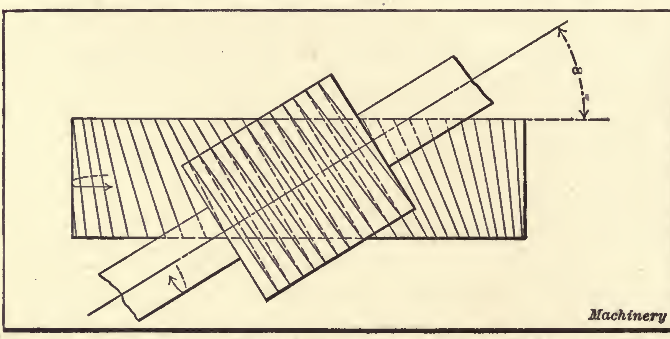


Fig. 14. Cutting a Left-hand Spiral Gear with a Right-hand Hob

if the hob revolves sixty-four times, sixty-four teeth will have passed by, but the blank is not in the same position as at the beginning.

In Fig. 17 *G* represents the position of the hob axis at the beginning of the cut and *H* the position of the hob axis after the hob has made sixty-four revolutions. This shows that the blank must make more than one revolution in this case. If we were cutting a left-hand spiral gear with a right-hand hob, as shown in Fig. 14, the blank would have to make less than one complete revolution for each sixty-four revolutions of the hob, the blank in this case being revolved in the opposite direction. It will thus be seen that when cutting a gear of the same "hand" as the hob, the table must revolve slightly faster than it would have to do when cutting a spur gear with the same number of teeth; but when the hob and the gear are of opposite "hand," the table must revolve more slowly than when cutting a spur gear. This

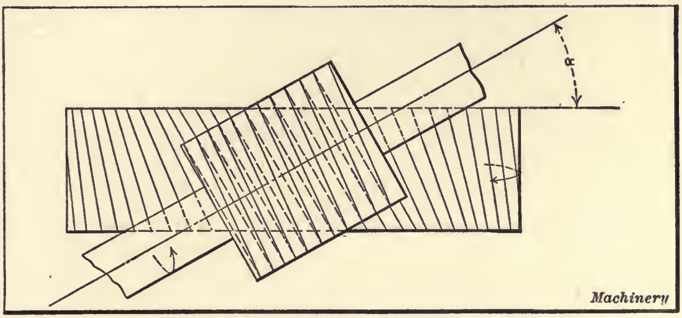


Fig. 15. Cutting a Left-hand Spiral Gear with a Left-hand Hob

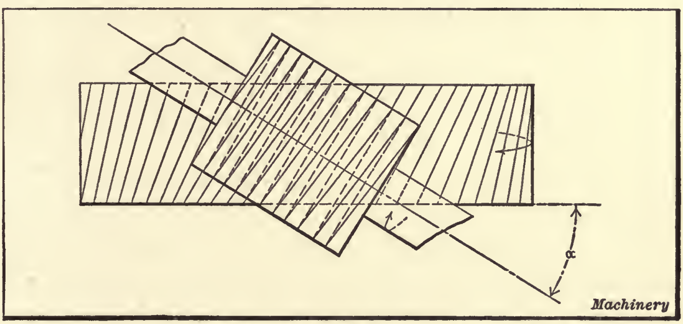


Fig. 16. Cutting a Right-hand Spiral Gear with a Left-hand Hob

has an important bearing upon the formula we are about to construct.

To gear the machine properly we must first find the ratio according to which the table is required to lag behind or lead ahead of its natural speed relative to the hob. In the first formula devised by the author for the hobbing of spiral gears, the ratio was arrived at by considering the number of revolutions made by the hob, while the table makes one full revolution.

The formula thus constructed for the No. 1 Farwell gear-hobbing machine is:

$$\frac{30 \times \text{No. of threads on hob}}{\text{No. of teeth} \pm [\,(\text{feed} \times \tan \text{of angle}) \div \text{circ. pitch}]} = \frac{P}{p}$$

This applies only to one particular machine. A later formula designed for the No. 3 Farwell machine, as shown in Fig. 12, considers the number of table revolutions required while the hob revolves a sufficient number of times to represent one revolution of the table, if we were cutting a spur gear:

$$\frac{20 \pm \dfrac{20}{\text{Pitch circumference} \div (\text{feed} \times \tan \text{of angle})}}{(3 \times \text{No. of teeth}) \div \text{No. of threads on hob}} = \frac{P}{p}$$

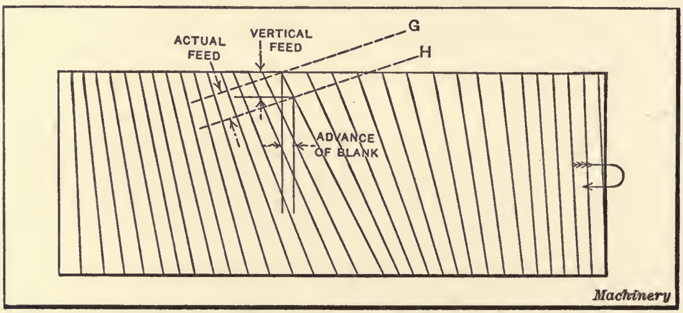


Fig. 17. Diagram showing Advance Required in Table Motion when Cutting a Left-hand Spiral Gear with a Left-hand Hob

Being called upon to derive another formula to be used for the new No. 3 Farwell universal hobbing machine, it occurred to the originator of these formulas, that a formula adapted to all hobbing machines would avoid much confusion. In the following is given the process by which such a formula was derived; the result is a simpler formula than any previously used.

Universal Formula. — The "lead" of a spiral gear is the axial length of the blank in which one spiral tooth makes a complete turn around the blank. Now, in hobbing a gear with a width of face exactly equal to the lead, it is evident that the blank must gain or lose one complete revolution as compared with the number of revolutions that would be made in cutting a spur gear with the same width of face and using the same feed per revolution of the blank. Assume that it is desired to cut a

30-tooth, 10-pitch, right-hand spiral gear of 45-degree angle, using a single-threaded right-hand hob and feeding $\frac{1}{32}$ inch across the face of the blank for each revolution of the blank.

The rule for finding the lead of a spiral gear is:

$$\text{Pitch circumference} \times \text{cot of tooth angle} = \text{lead.}$$

To get the pitch circumference, first find the pitch diameter; the rule for finding this in a spiral gear is:

> Pitch diameter of spur gear ÷ cos of tooth angle = pitch diameter of spiral gear with the same number of teeth and pitch.

A 30-tooth, 10-pitch spur gear would have a pitch diameter of 3 inches. Referring to a table of trigonometrical functions it will be found that the cosine of 45 degrees is 0.70711; then, $3 \div 0.70711 = 4.242$ inches, which is the pitch diameter of the spiral gear. Multiplying this by 3.1416 gives 13.3267 inches, which is the pitch circumference of the spiral gear. Since the cotangent of 45 degrees is exactly 1, multiplying by this gives the same quantity (13.3267 inches) as the lead.

The next step is to find how many times the blank must revolve while the hob feeds 13.3267 inches across its face. Since the feed is $\frac{1}{32}$ inch (0.03125) for each revolution, we can divide by 0.03125 or multiply by 32 to get the number of revolutions. This gives 426.454 revolutions. The table has been traveling faster in relation to the hob than would be the case in cutting a spur gear with the same number of teeth; in fact, the table has gained exactly one revolution on the hob. In other words, the table speed in cutting this spiral gear is to the table speed in cutting an equivalent spur gear as 426.454 is to 425.454. From this we may construct the following formula:

$$\frac{\text{Lead} \div \text{feed}}{(\text{Lead} \div \text{feed}) - 1} = \frac{\text{required table revolutions}}{\text{normal table revolutions}}$$

For a gear of opposite "hand" from that of the hob the sign would be changed to + in this formula. Use the − sign only when gear and hob are of the same "hand."

By adding a 426-tooth gear to the drivers and a 425-tooth gear to the driven gears in the regular combination used to cut a 30-tooth spur gear, we would get approximately the desired ratio, but for greater accuracy we can carry the figures to a few decimal places and factor:

$$\frac{42{,}645}{42{,}545} = \frac{8529}{8509} = \frac{3 \times 2843}{67 \times 127}$$

but 2843 is a prime number. We, therefore, try

$$\frac{4265}{4255} = \frac{853}{851}$$

but 853 is a prime number. We, therefore, try

$$\frac{4264}{4254} = \frac{2132}{2127} = \frac{4 \times 533}{3 \times 709}$$

but 709 is a prime number. Hence we must make another slight change and try again, remembering that whatever change is made in the numerator must be exactly duplicated in the denominator to maintain the ratio as nearly as possible. The dropping of all decimals would cause a very small error, but dropping them from one side only would cause a great error. We find upon trial that

$$\frac{426}{425} = \frac{2 \times 3 \times 71}{5 \times 5 \times 17}$$

Multiplying this with the change-gear combination ordinarily used to cut spur gears with 30 teeth, we have the gear combination required for any gear-hobbing machine used for cutting this gear. Thus, on the No. 3 Farwell universal hobbing machine, the spur-gear ratio for cutting 30 teeth is $\frac{30}{60}$, which multiplied by $\frac{2 \times 3 \times 71}{5 \times 5 \times 17}$ gives $\frac{3 \times 71}{5 \times 5 \times 17}$ and arranging this ratio in convenient gear sizes, we have:

$$\frac{24 \times 71}{40 \times 85} = \frac{\text{product of teeth of driving gears}}{\text{product of teeth of driven gears}}$$

It will be noted that the last operation before factoring was to divide by the feed. Should prime numbers be encountered repeatedly in trying to factor, it is possible to get altogether new figures to work with, by making a slight change in the feed and dividing into the lead again.

Having found the gears, set the feed for exactly $\frac{1}{32}$ inch per revolution, see that the table is revolving in the right direction, and tilt the hob spindle to bring the *thread* angle to 45 degrees and the machine is ready for business.

Recapitulation and General Remarks. — The general formula for gearing any hobbing machine for generating spiral gears is thus:

$$\frac{L \div F}{(L \div F) \pm 1} \times \frac{P}{p} = \frac{S}{s}$$

in which

L = lead of spiral;
F = feed per revolution;
P = product of driving gears for cutting spur gears with same number of teeth;
p = product of driven gears for cutting spur gears with same number of teeth;
S = product of driving gears for cutting spiral gears;
s = product of driven gears for cutting spiral gears.

Use + sign when gear and hob are of opposite "hand," and − sign when they are of the same "hand."

In cutting teeth at large angles it is desirable to have the hob the same hand as the gear, so that the direction of the cut will come against the movement of the blank, but at ordinary angles one hob will cut both right- and left-hand gears.

The actual feed of the cutter depends upon the angle of the teeth as well as on the vertical movement of the hob. This is obtained by dividing the vertical feed by the cosine of the tooth angle; thus:

$$\frac{0.03125}{0.70711} = 0.043 \text{ inch actual feed.}$$

The last computation need not be made except to see that we are not figuring on too heavy a cut, as it has nothing to do with the gearing of the hobbing machine. In setting up a hobbing machine for spiral gears, care should be taken to see that the vertical feed does not trip until the machine has been stopped or the hob has fed down clear of the finished gear. Should the feed stop while the hob is still in mesh with the gear and revolving at the ratio required to generate a spiral, the hob will cut into the teeth and spoil the gear.

Should the thread angle of the hob be exactly equal to the tooth angle of the spiral gear, and both hob and gear be the same "hand," the axis of the hob spindle will be at right angles to the axis of the gear. This is in conformity with the rule that when hob and gear are of the same "hand," the hob spindle is set at the tooth angle minus the thread angle of the hob. In cutting a spiral gear to take the place of a worm-wheel, it is possible to use the same hob that was used in cutting the worm-wheel. This would be a case where it is not necessary to tilt the hob spindle. Sometimes multiple-threaded hobs are used in order to make the thread angle approximately equal to the tooth angle, when it is desired to cut spiral gears with machines on which the hob spindle swivels through only a small angle.

Examples of Calculations. — As an example of the application of the formula given for finding the gears for spiral gear hobbing, assume that two spiral gears are to be cut on a gear-hobbing machine. Gear No. 1 has 30 teeth, 24.549-inch lead and a feed of $\frac{1}{24}$ inch. The change gears used on the machine for cutting a spur gear with 30 teeth have 48 (driving gear) and 60 (driven gear) teeth, respectively. The hob and gear are of the same "hand."

Gear No. 2 has 60 teeth, 49.098-inch lead and is cut with a feed of $\frac{1}{16}$ inch. The change gears used to cut a spur gear with 60 teeth, on this machine, have 48 and 40 teeth, for the driving gears, and 60 and 80 teeth, for the driven gears. The hob and gear are of the same "hand."

In the problems given the data are thus as follows:

	30-tooth Gear		60-tooth Gear
L	24.549	L	49.098
F	$\frac{1}{24}$	F	$\frac{1}{16}$
P	48	P	40×48
p	60	p	60×80

The same notation as in the formula just given is used.

Calculations for Thirty-tooth Gear. — By inserting the values given, we find that:

$$\frac{L \div F}{(L \div F) - 1} = \frac{589.176}{588.176}$$

The ratio written above can be simplified to the form $\frac{589}{588}$. Factoring, we have:

$$\frac{589}{588} = \frac{19 \times 31}{12 \times 49}$$

Now, multiply this value with the ratio of the gears for a 30-tooth spur gear:

$$\frac{19 \times 31}{12 \times 49} \times \frac{48}{60} = \frac{76 \times 31}{60 \times 49}$$

Having obtained the gears that should be used, we may now investigate what lead these gears will give. Apparently they will not give the exact lead desired, as we have used an approximate ratio instead of the exact one.

To prove, assume $F = \frac{1}{24}$ and solve for L.

$$\frac{L \div F}{(L \div F) - 1} = \frac{589}{588}$$

From this we find $L = 24.541$, which is very nearly equal to the required lead.

Calculations for Sixty-tooth Gear. — By proceeding in the same way for the 60-tooth gear we have:

$$\frac{L \div F}{(L \div F) - 1} = \frac{785.568}{784.568}$$

We then factor the fraction $\frac{785}{784}$, thus:

$$\frac{785}{784} = \frac{5 \times 157}{4 \times 196}$$

As 157 is a prime number, and gives too large a number of teeth for any of the gears in the train, we try $\frac{784}{783}$ which ratio is very nearly equivalent to that required.

$$\frac{784}{783} = \frac{49 \times 16}{29 \times 27}$$

Multiply this value with the ratio of the gears for a 60-tooth spur gear:

$$\frac{49 \times 16}{29 \times 27} \times \frac{40 \times 48}{60 \times 80} = \frac{49 \times 32}{29 \times 135}, \quad \text{or} \quad \frac{49 \times 32}{87 \times 45}$$

Possibly the 135-tooth gear is impracticable, on account of being too large, in which case the other combination must be tried.

If the lead resulting from the gears found is calculated in the same manner as in the previous case, we find that

$$L = 49.001.$$

Influence of Small Changes in the Ratio on the Lead. — It is interesting to note that a comparatively slight change in the ratio $\frac{L \div F}{(L \div F) - 1}$ makes a very decided change in the lead obtained. To illustrate, assume that in the first example given the ratio $\frac{589}{588} = 1.001701$ were changed to 1.002; let us see what effect this change would have on the lead obtained ($F = \frac{1}{24}$):

$$\frac{L \div F}{(L \div F) - 1} = 1.002.$$

If we solve for L in this equation we find $L = 20.875$, which is a very different lead from the one we wish to obtain.

Advantage of Differential Mechanism on Gear-hobbing Machines in Calculating Change Gears. — When generating helical gears on hobbing machines without a differential, the required ratio which combines index and feed gears must be calculated to a great many decimals, as otherwise a large error will result which will impair the accuracy of the gears. It frequently happens that the required ratio consists of prime numbers, especially when cutting right- and left-hand gears with one hob. To produce correct helical gears with their axes standing parallel to each other, the errors for the right- and left-hand spirals must be absolutely the same, otherwise there will not be a bearing on the whole length of the teeth. In fact, exactly the same conditions exist with helical gears as with spur gears. If, for instance, the teeth of one of two spur gears stood at an angle of only a few seconds with its axis, the bearing would be at one end of the teeth only.

Furthermore, if the hobbing machine has a differential, it is not necessary to have a right- and left-hand hob when cutting any angle up to 30 degrees; on the contrary, a higher efficiency is obtained when using only one hob for both right- and left-hand spirals. The reason for this is very simple; if there is any distortion in hardening, the right-hand hob will be different from the left-hand.

It has been mentioned before that the ratio must be calculated to several decimals when cutting the gears on machines without a differential. The belief of many mechanics that the ratios and errors obtained by formulas are alike for all hobbing machines, with or without differential mechanism, is entirely erroneous. There is a great difference between the two ratios. In the one case the ratio represents the value of the indexing and the helical movement, and the slightest change of the "driver," *viz.*, numerator, will cause a great error if the "driven," *viz.*, denominator, is not also changed in the same proportion. In the other case, *i.e.*, with the differential, the ratio obtained refers to the angle or helical movement only, and adds or subtracts itself automatically to or from the ratio of the indexing gears. The indexing gears required for cutting helical gears are given on a chart and can be read off the same as for spur gears. This is impossible without the differential. The difference between the two ratios is explained in the following example.

Example. — Gear, 48 teeth; 10 pitch; 20 degrees; $\frac{1}{16}$ inch feed per revolution of table.

Gear ratio of machine with differential for 20 degrees = 1.2052784.

If we deduct 1 from the third decimal which is 5, and omit the rest, we have 1.204 = ratio for 19 degrees 58 minutes 42 seconds; *i.e.*, 1 minute 18 seconds difference.

This shows how slight the error would be if we were to change the third decimal; in practice the change is made on the fifth decimal, and the error almost eliminated.

For the same pitch, number of teeth, angle and feed, the gear ratio for one of the machines without a differential equals 1.2517385. If here we were to deduct 1 from the third decimal and omit the rest, the result would be that instead of generating teeth the material would simply be milled off from the blank. This is explained as follows: Gear ratio for 20 degrees is 1.2517385. When deducting 1 from the third decimal we obtain 1.250, which is the spur-gear ratio.

The Schuchardt & Schütte gear-hobbing machines are provided with a differential which on the new type of machines is independent of the feed and indexing; in other words, when changing the number of teeth, or feed, or from right- to left-hand gear, no calculation is required. Thus the great advantage of the differential mechanism is that the helical movement is not disturbed whatever when the number of teeth is increased or decreased or the feed is changed. Suppose we intend to generate helical gears with 30, 40, 56 and 60 teeth, of which those having 40 and 60 teeth are left-hand, and those having 30 and 56 teeth are right-hand; the spiral angle is 15 degrees; the pitch is 10. The material is supposed to be cast iron; therefore $\frac{1}{16}$-inch feed per revolution of the table would be selected as the proper one. All the gears are to be cut with one right-hand 10-pitch hob. In calculating the change gears used when generating these gears on the Schuchardt & Schütte machine but a few minutes will be required, the following formula being used:

$$\frac{\text{Constant} \times \text{sine of angle} \times \text{pitch}}{1} = \text{ratio}.$$

$$\frac{0.3524 \times 0.25882 \times 10}{1} = \frac{912}{1000} = \frac{19 \times 48}{20 \times 50} = \frac{\text{driving gears}}{\text{driven gears}}$$

On machines not provided with a differential mechanism, every gear of the same pitch, with only a different number of teeth, must be calculated for separately, and the slightest change in the feed will require a separate calculation. A change in the formula must also be made, if right- and left-hand gears with the same number of teeth are cut with one hob.

The differential is also of great importance when cutting worm-gears with a taper hob. Worm-gears for worms with multiple threads ought to be generated with taper hobs if high efficiency is required.

CHAPTER V

HOBS FOR SPUR AND SPIRAL GEARS

Hobbing vs. Milling of Gears. — The adverse criticism of the gear-hobbing process has been the cause of many interesting investigations, and one of the most important of these has been the comparative study of the condition of the surfaces produced by the hob and by the rotary cutter. In making such a comparative study, it is necessary that the investigator possess the required practical knowledge, and also that he be willing to admit a point, even though his favorite processes may suffer by the comparison.

Feed Marks Produced by Rotating Milling Cutters. — While both the gear-hobbing machine and the automatic gear cutter use rotating cutting tools, the operations cannot be placed on a common basis and considered as similar milling operations, although they may, to a certain extent, be compared as such. In comparing the quality of the surfaces produced by the two processes, consider first the milled surface produced by an ordinary rotary cutter. This surface has a series of hills and hollows at regular intervals, the spacing between these depending upon the feed per revolution of the cutter, and the depth on both the feed and the diameter of the cutter. The ridges are more prominent when coarse feeds and small diameter cutters are used. These feed marks are the result of the convex path of the cutting edge and the slight running out of the cutter, which is inevitable in all rotary cutters with a number of teeth. As is well known to those familiar with milling operations, the spacing of the marks does not depend on the number of teeth in the cutter. Theoretically, it should depend on this number, but as it is practically impossible to get a cutter which will run absolutely true with the axis of rotation, only one mark is produced for each revolution, and, hence, the spacing becomes equal to the feed per revolution. The eccentricity of the cutter with the axis of rotation is, therefore, the factor which, together with the diameter of the cutter and the feed per revolution, determines the quality of the surface, other conditions being equal.

The depth of the hollow produced by the high side of the revolving cutter is equal to the height or rise of a circular arc, the radius of which equals the radius of the cutter, and the chord of which equals the feed per revolution. (See Fig. 1.) The length of the chord or the feed per revolution may be expressed:

$$F = 2 \times \sqrt{2HR - H^2}$$

in which F = feed per revolution;
H = height of arc;
R = radius of cutter.

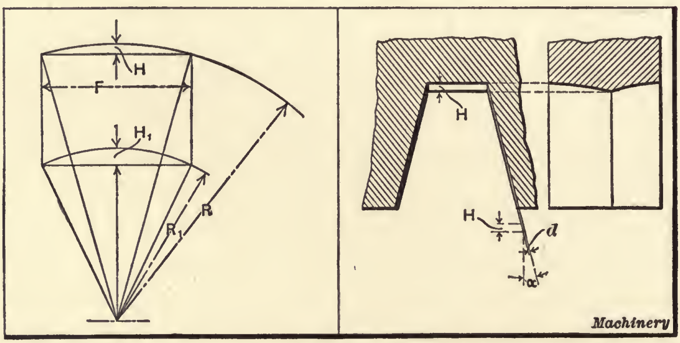


Fig. 1. Diagram illustrating the Relation between Feed, Diameter of Cutter and Depth of Feed Marks

Fig. 2. Diagram for finding Depth of Feed Marks on Side of Tooth cut by Milling Cutter

Since H^2 is a very small quantity, it may be discarded in the expression, which is then simplified to read:

$$F = 2 \times \sqrt{HD}$$

in which D = diameter of cutter.

Transposing this expression, we obtain $H = \frac{F^2}{4D}$, which is an approximately correct expression of the depth of the hollows produced by milling. As an example, take an 8-pitch rack cutter, with straight rack-shaped sides, 3 inches in diameter, milling with a feed per revolution of 0.1 inch. The depth of the feed marks at the bottom of the cut will be equal to:

$$\frac{(0.1)^2}{4 \times 3} = 0.00083 \text{ inch.}$$

The working surface of the tooth, however, is produced by the side of the cutter, as illustrated in Fig. 2, and the depth of the feed marks is normal to the surface, and is expressed as:

$$d = H \times \sin \alpha$$

in which d = depth of the feed marks on the side of the tooth, and α the angle of obliquity. In the example given, the depth d would equal 0.00021 inch, for a $14\frac{1}{2}$-degree involute tooth.

The depth of the feed marks is inversely proportional to the diameter of the cutter, and is, therefore, greater at the point of

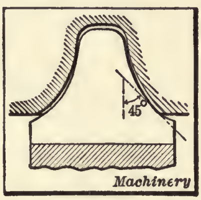


Fig. 3. Angle which limits the Feed

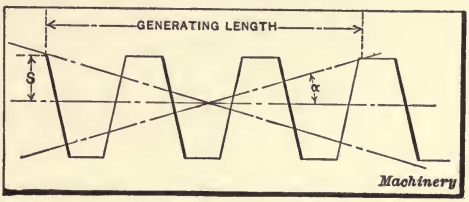


Fig. 4. Diagram for deducing Formulas for analyzing Action in Gear Hobbing Machine

the tooth than at the root. In the example given the depth would be 0.00025 inch at the extreme point of the rack tooth. It is thus apparent that the quality of the surface at any position along the tooth from the root to the point depends upon the diameter and form of the cutter and the feed per revolution.

In Fig. 3 is shown the outline of a No. 6 standard $14\frac{1}{2}$-degree involute gear cutter. This outline, at the point close to the end of the tooth of the gear, is a tangent inclined at an angle of 45 degrees, as indicated. Hence, the depth of the revolution marks is:

$\frac{(0.1)^2}{4 \times 2.5} \times \sin 45^\circ = 0.000707$ inch, instead of 0.00024 inch, as in the case of the straight rack tooth. It is evident that to produce an equal degree of finish with that left by the rack cutter, the feed must be considerably less for a No. 6 involute gear cutter than for the rack cutter. In Fig. 5 is shown the full range of cutter profiles from Nos. 1 to 8, with the angle of the tangent in each case which determines the quality of the surface under equal conditions of feed and diameter of cutter.

If the depth of the feed marks is used as the determining factor in comparing the condition of the surfaces produced by a series of cutters, it is evident that if the surface produced by the rack cutter is taken as a standard, the feed for cutting a pinion must be considerably less than the feed used for cutting gears with a large number of teeth. In fact, if a rack cutter is fed 0.100 inch per revolution, a No. 8 standard involute gear cutter should not be fed more than 0.055 inch per revolution to produce an equally good surface. The feed is proportional to the square

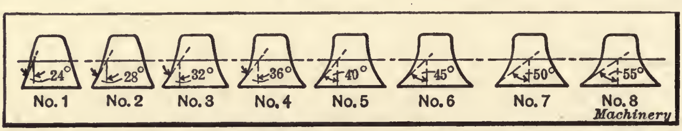


Fig. 5. Angles limiting the Feed for 14½-degree Standard Gear Cutters

root of the reciprocal of the sine of the angle of the limiting tangent.

If we assume the accuracy of the surface left by the straight-sided rack cutter as equal to 100 per cent, then the relative feeds required for cutting gears with any formed cutter can be calculated. This has been done, and the results are shown plotted in curve *A*, in Fig. 6. This curve is based on an equal depth of the feed marks for the full range of numbers of teeth in the gears. If, on the other hand, the surfaces left by the cutter for a given feed per revolution are compared, the depth of the feed marks will vary with the sine of the angle of the limiting tangent, and taking the straight-sided rack cutter as a basis, the relative accuracy of the surfaces is inversely proportional to the sine of the angle, and is plotted in curve *B*, in Fig. 6.

Comparison between Surfaces Produced by Milling and Hobbing. — A relation has now been established between the quality of the surface and the permissible feeds for cutters for cutting gears with any number of teeth. We will now consider the condition of the surface produced by a hob in a gear-hobbing machine. The hob is made with straight-sided rack-shaped teeth and with sides of a constant angle, and is used to produce gears with any number of teeth. We may therefore assume that it is cutting under the conditions governing the rack cutter, as just explained, the surface produced being considered merely as a milled surface. If this assumption be correct, then the quality of the surface produced by a hob, whether cutting a gear of twelve teeth or of two hundred teeth, will be the same for a given feed,

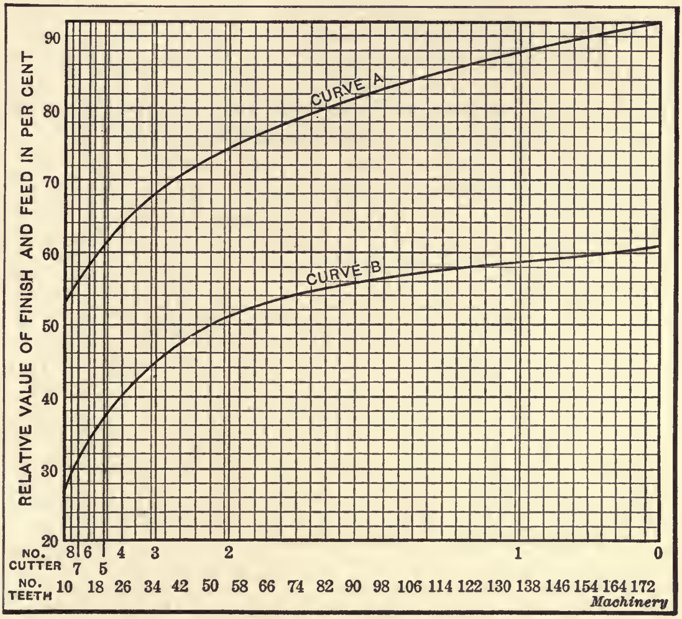


Fig. 6. Diagrams showing the Relation between Feed, Finish, and Number of Teeth when cutting Gears with Formed Gear Cutters

and the same relation exists between the hob and any formed cutter that exists between the rack cutter and any formed cutter; hence, curves *A* and *B*, in Fig. 6, may be assumed to show the permissible feeds and the quality of the surfaces produced by formed cutters when compared with the surfaces produced by a hob, provided the surfaces are considered merely as milled surfaces. However, a condition enters in the case of the hob which has no equivalent in the case of the formed milling cutter, and this influences the condition of the surface. This condition is the distortion of the hob teeth in hardening which causes them to mar the surface of the tooth by "side swiping," producing a rough surface. The eccentricity of the hob with the axis of rotation also has a different effect on the surface than in the case of a formed gear cutter. The effect is shown in a series of flats running parallel with the bottom of the tooth, if excessive; if the eccentricity is small, the effect will merely be to round the top of the tooth. These inaccuracies, however, can be taken care of in a number of ways.

Comparison of Output. — For reasons not connected with the quality of the surface, the hob may be worked at a greater cutting

Comparison of Time Required for Cutting Gears on Automatic Gear-cutting Machines and Hobbing Machines

Number of Teeth	Automatic Gear Cutters		Gear-hobbing Machines	
	Feed, Inches	Time, Minutes	Feed, Inches	Time, Minutes
32	0.022	15	0.050	6.5
31	0.020	19	0.050	9
24	0.024	22	0.050	6
17	0.020	8	0.050	4
17	0.020	17.5	0.050	5
16	0.018	10.5	0.050	7
13	0.013	6.25	0.050	6

speed and feed than a rotary cutter, when cutting from the solid, the reason being due to the generating action of the hob which results in the breaking of the chips. This preserves the cutting edges and reduces the heating effect of the cut, and explains why the hob may give such good results as compared with a rotary cutter in the matter of output. It is possible to get good results in the general run of work in the hobbing machine in one-third to one-half of the time required in an automatic gear cutter. The accompanying table gives the results obtained on automobile transmission gears with automatic gear-cutting machines and hobbing machines. If anything, the conditions under which the comparisons were made favored the automatic machines. Here, of course, spur gears are considered, but the relative advantages are still greater in the case of spiral gears. The speed of the cutter in all cases was 120 revolutions per minute, except in the case of the 13-tooth pinion, when the speed was raised to 160 R.P.M. to increase the output. The hob was run at a speed of 105 R.P.M. in all cases. The hob and cutters were of practically the same diameter. The results were obtained in producing an ordinary day's work and clearly indicate the advantage of the hobbing process over the milling process, when the quality of the tooth surface alone is considered, on the basis that both processes produce a milled surface.

The Tooth Outline. — Going further into the subject, we will take up the question of the tooth outline. The tooth of a gear milled with an ordinary milling cutter must be, or at least is expected to be, a reproduction of the outline of the cutter, and since each cutter must cover a wide range of teeth, the outline is not theoretically correct, except for one given number of teeth in the range. Theoretically speaking, the outline of the hobbed tooth may be considered as a series of tangents, the tooth surface being composed of a series of flats parallel with the axis of the gear. To show the significance of these flats, assume, for example, that a gear with thirty-two teeth is cut with a standard hob, 8 pitch, 3 inches in diameter, having twelve flutes. The length of the portion of the hob that generates the tooth surface is $2\,S \div \tan\alpha$, where α is the pressure angle, as indicated in Fig. 4. The number of teeth following in the generating path is:

$$\left(\frac{2\,S}{\tan\alpha} \div \text{circular pitch}\right) \times \text{number of gashes.}$$

In this case the generating length is approximately 0.96 inch, and there are thirty teeth in the generating path. The flats of those parts of the tooth outline which each of the hob teeth form vary in width along the curves. They are of minimum width at the base line and of maximum width at the point of the tooth. The width of the flats at the pitch circle is proportional to the number of teeth in the gear, the number of gashes in the hob and the pressure angle. The angle β, to the left in Fig. 7, which is the angle between each flat, is proportional to the number of teeth in the gear and the number of gashes in the hob. In the example given it is:

$$\beta = \frac{360 \times \frac{0.96}{3.1416 \times 4}}{30} = 0.91 \text{ degree, or } 55 \text{ minutes.}$$

The Width of Flat Produced. — The width a of the flat at the pitch line is equal to twice the tangent of one-half β times the

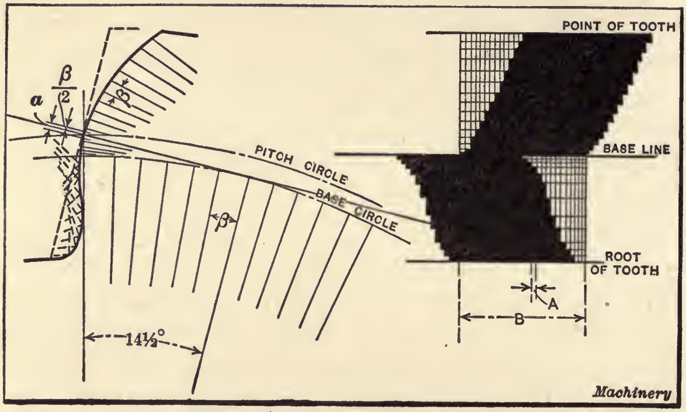


Fig. 7. Relative Width and Position of Flats produced by Gear Hobbing Machines. A indicates Feed per Each Generating Tooth; B, Feed per Revolution of Blank

length of the pressure line between the point of tangency with the base line and the pitch point, and is:

$$a = 2 \tan \tfrac{1}{2} \beta \times \tan 14\tfrac{1}{2}^\circ \times 2 = 0.0081 \text{ inch.}$$

This is not a flat that could cause serious trouble. As in the case of the feed marks, it is not the width of the flat alone that is to be considered, but the depth must be taken into account; in fact, the quality of the surface may be spoken of as the ratio of the depth to the length of the flat. The depth of the flat is the rise or height of the arc of the involute and is approximately proportional to the versed sine of the angle $\frac{1}{2} \beta$, and with the pitch assumed in the example given would be 0.000015 inch. It is difficult to conceive of any shock caused by this flat, as the gear teeth roll over each other. The action of the hob and gear in relation to each other further modifies the flat by giving it a crowning or convex shape. In fact, the wider the flat the more it is crowned. This explains the fact that hobs with a few gashes produce teeth of practically as good shape as hobs with a large number of gashes. It is desirable, therefore, to use hobs with as few gashes as possible, because from a practical point of view the errors of workmanship and those caused by warping in hardening increase with the number of flutes.

A peculiar feature of the hobbed tooth surface is shown to the right in Fig. 7, which illustrates the path on a tooth produced by a hob in one revolution. In fact, there are two distinct paths, the first starting at the point of the tooth and working down to the base line, the cutting edges of the hob tooth then jumping to the root of the tooth and working up to the base line, producing the zigzag path shown.

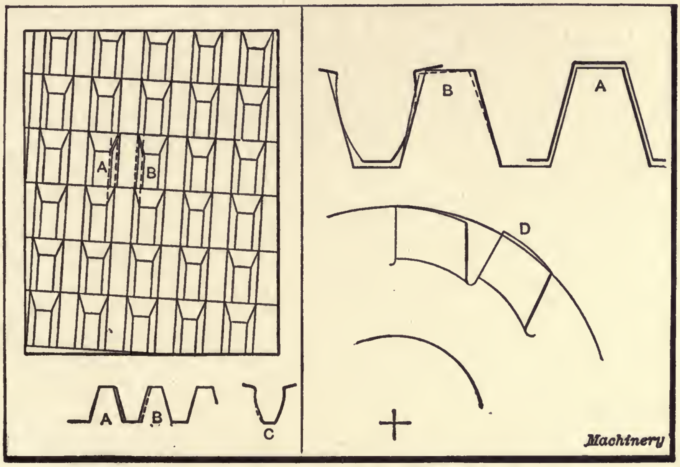
Summary of the Preceding Comparative Study. — That the flats so commonly seen in the results obtained from the hobbing machine are not due to any faults of the process that cannot be corrected, but are due to either carelessness on the part of the operator in setting up the machine without proper support to the work, or to the poor condition of the hob or machine, and that nearly all cases of flats can be overcome by the use of a proper hob, may be assumed as a statement of facts. When the hobbing machine will not give good results, the hob is in nearly all cases at fault. If a gear is produced that bears hard on the point of the teeth, has a flat at the pitch line or at any point along the face of the tooth, do not think that the process is faulty in theory, or that the machine is not properly adjusted, or that the strain of the cut is causing undue torsion in the shafts, or that there is backlash between the gears in the train connecting the work and the hob; these things are not as likely to cause the trouble as is a faulty hob.

After an experience covering all makes of hobbing machines, the author has come to the conclusion that the real cause of the trouble in nearly every case is a faulty hob. Machine after machine has been taken apart, overhauled and readjusted, and yet no better results have been obtained until a new and better hob was produced. The faults usually met with in hobs will be referred to in the following, together with the means for getting the hob into a good working condition. It is not desired in any way to disparage the formed cutter process in favor of the hobbing process, but simply to state the facts as they appear. In every case, practice seems to substantiate the conclusions arrived at.

Hobs for Spur and Spiral Gears. — The success of the hobbing process for cutting teeth in spur and spiral gears depends, as stated, more upon the hob than upon the machine, and at the present stage of development the hob is the limiting factor in the quality of the product. It is well known that hobs at the present time are far from being standardized, and that the product will not be interchangeable if the hob of one maker is substituted for that of another; in fact, the using of two hobs from the same maker successively will sometimes result in the production of gears which will not interchange or run smoothly. This is not a fault of the hobbing process, but is due to the fact that the cutter manufacturers have not given the question of hobs the study it requires. This is also the reason why there are so many complaints about the hobbing machine. Nevertheless, with the proper hob the hobbing machine is the quickest method of machining gears that has ever been devised. Its advantage lies in the continuous action and in the simplicity of its mechanism. There is no machine for producing the teeth of spur gears that can be constructed with a simpler mechanism, and even machines using rotary cutters are more complicated if automatic.

Hobs with Few Teeth Give Best Results. — The ideal form of hob, theoretically speaking, would be one that had an infinite number of cutting teeth. In practice, however, a seemingly contradictory result is obtained, as hobs with comparatively few teeth give the best results. The reasons for this are due to purely practical considerations. Strictly speaking, a theoretical tooth curve is no more possible when the tooth is produced by the hobbing process than when produced by the shaper or planer type of generator, but for all practical purposes, the curve generated under proper working conditions is so nearly correct as to be classed as a theoretical curve. If this result is not often met with under ordinary working conditions, it is due to the fact that the hob is not as good as present practice is able to make it.

Causes of Defects in Hobbed Gears. — In order to obtain, as far as is theoretically possible, a proper curve and not a series of flat surfaces, the teeth of the hob must follow in a true helical path. In ninety-nine cases out of one hundred the hob is at fault when a series of flats is obtained instead of a smooth curved tooth face. It is the deviation of the teeth of the hob from the helical path that is at the root of most hobbing machine troubles. There are several causes for the teeth being out of the helical path: The trouble may have originated in the relieving or forming of the teeth; the machine on which this work has been done



Figs. 8 and 9. Distortion of Hob Teeth and its Effect

may have been too light in construction, so that the tool has not been held properly to its work, and has sprung to one side or another causing thick and thin teeth in the hob; a hard spot may have been encountered causing the tool to spring; the gashes may not have been properly spaced, or there may have been an error in the gears on the relieving lathe influencing the form; the hob may also have been distorted in hardening; it may have been improperly handled in the fire or bath, or it may have been so proportioned that it could not heat or cool uniformly; the grinding after hardening may be at fault; the hole may not have been ground concentric with the form, thus causing the teeth on one side of the hob to cut deeper than on the other.

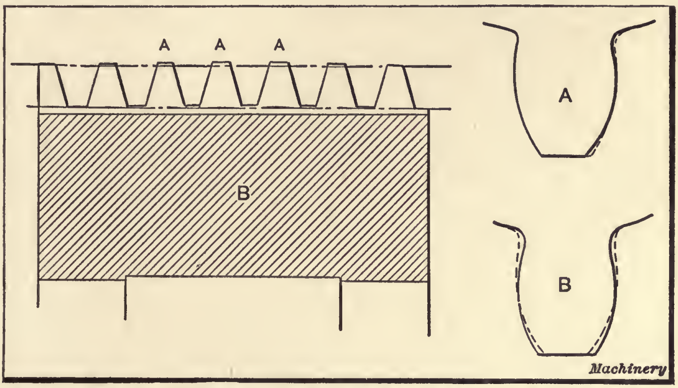
Any one or a combination of several of these conditions may have thrown the teeth out of the true helical path.

Fig. 8 illustrates the difficulty of thick and thin teeth. The tooth *A* is too thick and *B* is too thin, the threading tool having sprung over from *A* and gouged into *B*. Fig. 8 also shows a developed layout of the hob. At *C* is shown the effect that thick and thin teeth may have on the tooth being cut in the gear — that of producing a flat on the tooth. This flat may appear on either side of the tooth and at almost any point from the root to the top, depending upon whether the particular hob tooth happens to come central with the gear or not. If it does come central or nearly so, it will cause the hob to cut thin teeth in the blank. The only practical method to make a hob of this kind fit for use is to have it re-formed.

In Fig. 9 is shown at *A* the result of unequal spacing of the teeth around the blank. Owing to the nature of the relief, the unequal spacing will cause the top of the teeth to be at different distances from the axis of the hob. This would produce a series of flats on the gear tooth. One result of distortion in hardening is shown at *B*, Fig. 9, where the tooth is canted over to one side so that one corner is out of the helical path. This defect also produces a flat and shows a peculiar under-cutting which at first is difficult to account for. Sometimes a tooth will distort under the effects of the fire in the manner indicated at *D*, Fig. 9.

These defects may be avoided by proper care and by having the steel in good condition before forming. The blank should be roughed out, bored, threaded, gashed and then annealed before finish-forming and hardening. The annealing relieves the stresses in the steel due to the rolling process.

The proportions of the hob have a direct effect on distortion in hardening. This is especially noticeable in hobs of large diameter for fine pitches. Fig. 10 shows the results obtained in hardening a 4-inch hob, 10 pitch, with $1\frac{1}{4}$-inch hole. There is a bulging or crowning of the teeth at *A*. This is accounted for by the fact that the mass of metal at *B* does not cool as quickly as that at the ends. Consequently, when the hob is quenched, the ends and outer shell cool most quickly and become set, preventing the mass at *B* from contracting as it would if it could come in direct contact with the cold bath and cool off as quickly as the rest of the metal. The effect of this distortion on the shape of the gear teeth is indicated in Fig. 11, where the tooth *A* is unsymmetrical in shape due to the fact that the teeth near the center of the hob cut deeper into the blank, under-cutting the tooth on one side and thinning the point. This effect is produced when the gear is centered near the ends of the hob. If the gear is centered midway of the length of the hob, the tooth



Figs. 10 and 11. Distortion of Hobs and Result on the Shape of the Teeth

shape produced is as shown at *B*, Fig. 11. This tooth is thick at the point and under-cut at the root.

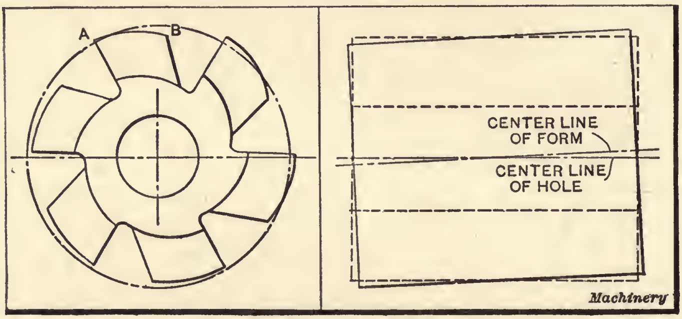
A hob in this condition makes it impossible to obtain quiet running gears. In this case, it would be useless to anneal and re-form the hob, as the same results would be certain to be met with again, on account of the proportions of the hob. Hence, defects of this kind are practically impossible to correct, and the hob should either be entirely remade or discarded.

Figs. 12 and 13 show in an exaggerated manner two common defects due to poor workmanship. In Fig. 12 the hole is ground out of true with the outside of the tooth form. The hole may run either parallel with the true axis of the hob, or it may run at an angle to it, as seen in Fig. 13.

The effect of the first condition is to produce a tooth shaped like that shown by the full lines at *B* in Fig. 11, and the effect of that in Fig. 13 is about the same, except that the hob will cut thin teeth when cutting to full depth. Gears cut by either hob will lock with meshing gears, and instead of smooth rolling, the action will be jerky and intermittent.

Gashes which originally were equally spaced may have become unequally spaced by having more ground off the face of some teeth than of others. The greater the amount of relief, the more particular one must be in having the gashes equally spaced.

Grinding to Correct Hob Defects. — These various faults may be corrected to a greater or less extent in the following manner:



Figs. 12 and 13. Hobs with the Center Hole out of True with the Outside of the Tooth Form

Place the hob on a true arbor and grind the outside as a shaft would be ground; touch all of the teeth just enough so that the faintest marks of the wheel can be seen on the tops. The teeth that are protruding and would cause trouble will, of course, show a wide ground land, while on those that are low, the land will be hardly visible. Now grind the face of each tooth back until the land on each is equal. This will bring all the teeth to the same height and the form will run true with the hole. To keep the hob in condition so that it will not be spoiled at the first re-sharpening, grind the backs of the teeth, using the face as a finger-guide, the same as when sharpening milling cutters, so as to remove enough from the back of each tooth to make the distance AB, Fig. 12, the same on all the teeth. Then, when sharpening the teeth in the future, use the back of the tooth as a finger-guide. If care is taken, the hob will then cut good gears as long as it lasts. It is poor practice to use the index head when sharpening hobs, because the form is never absolutely true with the hole, and unless the hob has been prepared as just described, there is no reliable way to sharpen it. If the hob, after having been prepared as described, is sharpened on centers by means of indexing, it will be brought back to the original condition.

The defect shown in Fig. 13 is corrected in the same manner. The gash when so ground will not be parallel with the axis in a straight-fluted hob, nor will it be at an exact right angle with the thread helix in a spiral-fluted hob, because the teeth at the right-hand end are high while those at the left-hand end are low, and the amount that must be ground off the faces of the hob teeth will be greater at one end than at the other. The angle will be slight, however, and of no consequence.

Shape of Hob Teeth. — The first thing that is questioned when a hob does not produce smooth running gears is the shape of the hob tooth. The poor bearing obtained when rolling two gears together would, in many cases, seem to indicate that the hob tooth was of improper shape, but in nearly every case the trouble is the result of one or more of the defects already pointed out.

Theoretically, the shape of the hob tooth should be that of a rack tooth with perfectly straight sides. This shape will cut good gears from thirty teeth and up, in the $14\frac{1}{2}$-degree involute system, but gears under thirty teeth will have a reduced bearing surface as a result of under-cutting near the base circle, which increases as the number of teeth grows smaller. The shape produced by such a hob, if mechanically perfect, would be a correct involute, and the gears should interchange without difficulty. In order that the beginning of contact, however, may take place without jar, the points of the teeth should be relieved or thinned, so that the contact takes place gradually, instead of with full pressure. This is accomplished by making the hob tooth thicker at the root, starting at a point considerably below the pitch line. This is illustrated in the upper portion of Fig. 14, which shows the standard shape adopted by the Barber-Colman Co. The shape of the tooth produced is also shown. The full lines show the shape generated, and the dotted, the lines of the true involute. The amount removed from the points is greater on large gears and less on small pinions, where the length of contact is none too great even with a full-shaped tooth, and where any great reduction must be avoided. This shape of hob tooth does not, however, reduce under-cutting on small pinions. The fact that hobbed gears have under-cut teeth in small pinions, while those cut with rotary cutters have radial flanks with the curve above the pitch line corrected to mesh with them is the reason why

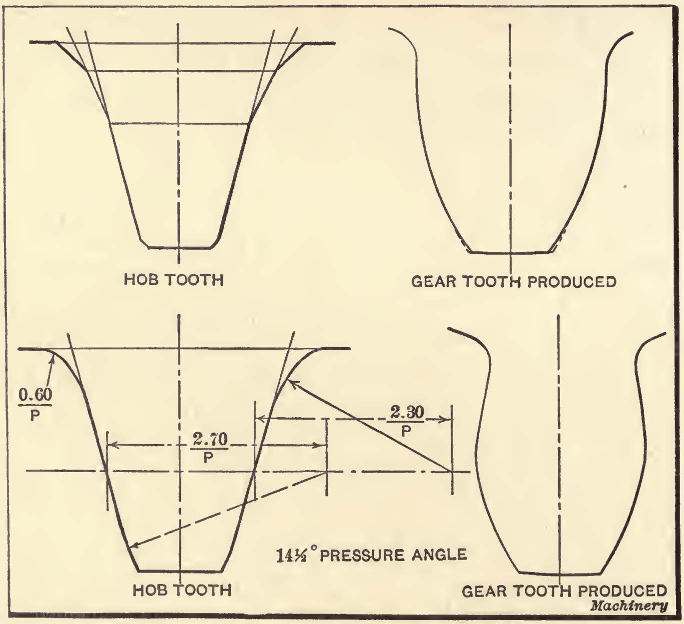


Fig. 14. Forms of Hob Teeth and Gear Teeth Produced

hobbed gears and those cut with rotary cutters will not interchange.

In the lower part of Fig. 14 is shown a hob tooth shape which will produce teeth in pinions without under-cutting, the teeth, instead, having a modified radial flank. The radii of the correction curves are such that the gear tooth will be slightly thin at the point to allow an easy approach of contact. The shape shown is approximately that which will be produced on hobs, the teeth of which are generated from the shape of a gear tooth cut with a rotary cutter, which it is desired to reproduce.

Diameters of Hobs. — The diameters of hobs is a subject which has been much discussed. Many favor large hobs because the larger the hob the greater the number of teeth obtainable. This, however, has already been shown to be a fault, because the greater is the possible chance of some of the teeth being distorted. For the same feed, the output of a small hob is greater, because of being inversely proportional to the diameter. The number of teeth cut is directly proportional to the number of revolutions per minute of the hob. The number of revolutions depends on the surface speed of the hob; therefore, the small hob will produce more gears at a given surface speed.

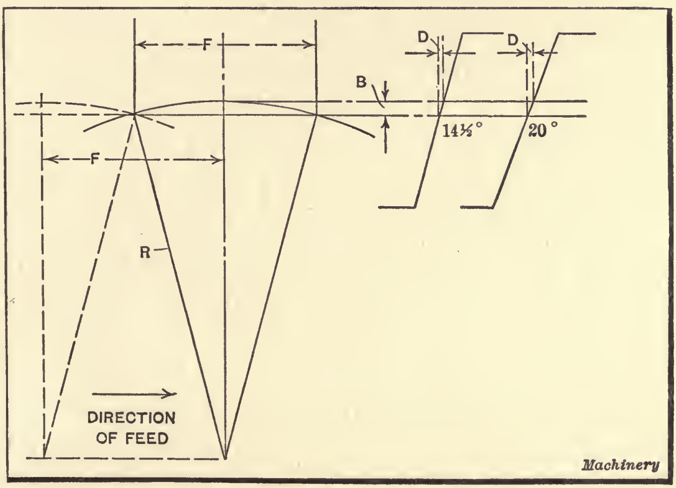


Fig. 15. Illustrating Effect of Feed in Hobbing

It may be argued that, on account of the large diameter, the large hob can be given a greater feed per revolution of the blank than the smaller hob, for a given quality of tooth surface. This argument is analyzed in Fig. 15. Let R be the radius of the hob and F the feed of the hob per revolution of the blank. Then B may be called the rise of feed arc.

Since the surface of the tooth is produced by the side, the actual depth of the feed marks is D, which depends on the angle of the side of the tooth, the depth being greater for a 20-degree tooth than it would be for a $14\frac{1}{2}$-degree tooth for the same amount of feed. The relations between F, R, and B may be expressed as follows:

$$F = 2\sqrt{2RB - B^2}$$

Since B is a very small fractional quantity, B^2 would be much smaller and can, therefore, be disregarded, giving the very simple approximate formula $F = 2\sqrt{2RB}$. A rise of 0.001 inch would mean a depth D of about 0.00025 inch on a $14\frac{1}{2}$-degree tooth. The allowable feed is 0.126 inch for a 4-inch hob and 0.108 inch for a 3-inch hob for a 0.001 inch rise. The curve in

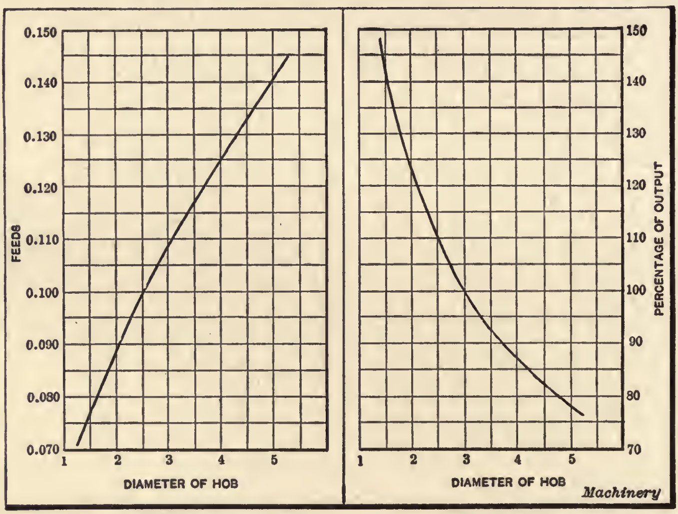


Fig. 16. Diagram showing Comparative Feeds for Hobs of Various Diameters

Fig. 17. Diagram showing Comparative Output of Hobs of Various Diameters based on 3-inch Hob

Fig. 16 shows the feeds for this rise for various hob diameters. Fig. 17 shows a curve based on a 3-inch hob that shows the comparative output for an equal rise. This curve shows that the smaller hob is superior in matter of production.

The larger hobs are also more liable to distortion in hardening and they do not clear themselves as well as the smaller ones when cutting; consequently, they need a greater amount of relief. Large hobs also require a greater over-run of feed at the start of the cut. When cutting spiral gears of large angles this greatly reduces the output, as the greater amount of feed required before the hob enters to full depth in the gear is a pure waste.

The question of whether the gashes or flutes should be parallel with the axis or at right angles to the thread helix has two sides. From a practical point of view, it appears to make very little difference in the results obtained in hobs of small pitch and angle of thread. In hobs of coarse pitch, however, the gashes should undoubtedly be normal to the thread. The effect of the straight gash is noticed when cutting steel, in that it is difficult to obtain a smooth surface on one side of the tooth, especially when cutting gears coarser than 10 pitch. What has been said in the foregoing, however, applies equally to straight and spirally fluted hobs.

CHAPTER VI

CALCULATING THE DIMENSIONS OF WORM GEARING

The present chapter contains a compilation of rules for the calculation of the dimensions of worm gearing, expressed with as much simplicity and clearness as possible. No attempt has been made to give rules for estimating the strength or durability of worm gearing, although the question of durability, especially, is the determining factor in the design of worm gearing. If the worm and wheel are so proportioned as to have a reasonably

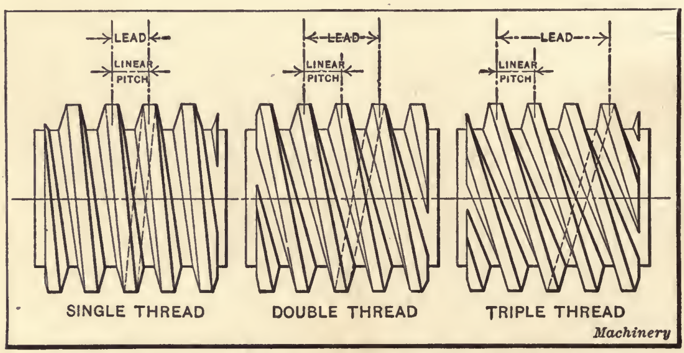


Fig. 1. Diagram showing Relation between Lead and Pitch

long life under normal working conditions, it may be taken for granted that the teeth are strong enough for the load they have to bear. No simple rules have ever been proposed for proportioning worm gearing to suit the service it is designed for. Judgment and experience are about the only factors the designer has for guidance. In Europe a number of builders are regularly manufacturing worm drives, guaranteed for a given horsepower at a given speed. Reference to the durability and power transmitting properties of worm gearing will be made in a following chapter.

Definitions and Rules for Dimensions of the Worm. — In giving names to the dimensions of the worm, there is one point in which there is sometimes confusion. This relates to the distinction between the terms "pitch" and "lead." In the following we will adhere to the nomenclature indicated in Fig. 1. Here are shown three worms, the first single-threaded, the second double-threaded, and the last triple-threaded. As shown, the word "lead" is assumed to mean the distance which a given thread advances in one revolution of the worm, while by "pitch," or more strictly, "linear pitch," we mean the distance between the centers of two adjacent threads. As may be clearly seen, the lead and linear pitch are equal for a single-threaded worm.

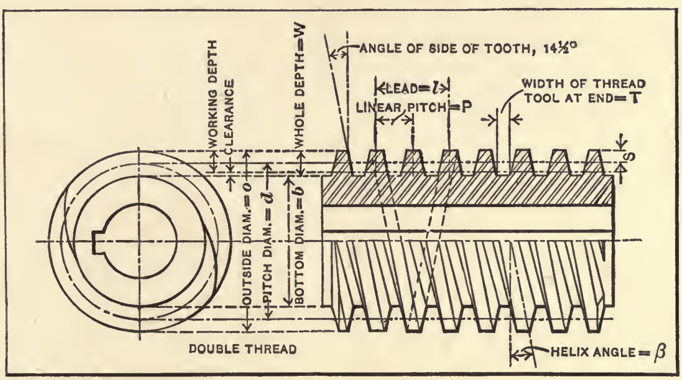


Fig. 2. Nomenclature used for Worm Dimensions

For a double-threaded worm the lead is twice the linear pitch, and for a triple-threaded worm it is three times the linear pitch. From this we have:

Rule 1. To find the lead of a worm, multiply the linear pitch by the number of threads.

It is understood, of course, that by the number of threads is meant not the number of threads per inch, but the number of threads in the whole worm — one, if it is single-threaded, four, if it is quadruple-threaded, etc. Rule 1 may be transposed to read as follows:

Rule 2. To find the linear pitch of a worm, divide the lead by the number of threads.

The standard form of worm thread, measured in an axial section, as shown in Fig. 2, has the same dimensions as the standard form of involute rack tooth of the same linear or circular pitch. It is not of exactly the same shape, however, not being rounded at the top, nor provided with fillets. The thread is cut with a straight-sided tool, having a square, flat end. The sides have an inclination with each other of 29 degrees, or $14\frac{1}{2}$ degrees with the center line. The following rules give the dimensions of the teeth in an axial section for various linear pitches. For nomenclature, see Fig. 2.

Rule 3. To find the whole depth of the worm tooth, multiply the linear pitch by 0.6866.

Rule 4. To find the width of the thread tool at the end, multiply the linear pitch by 0.31.

Rule 5. To find the addendum or height of worm tooth above the pitch line, multiply the linear pitch by 0.3183.

Rule 6. To find the outside diameter of the worm, add together the pitch diameter and twice the addendum.

Rule 7. To find the pitch diameter of the worm, subtract twice the addendum from the outside diameter.

Rule 8. To find the bottom diameter of the worm, subtract twice the whole depth of tooth from the outside diameter.

Rule 9. To find the helix angle of the worm and the gashing angle of the worm-wheel tooth, multiply the pitch diameter of the worm by 3.1416, and divide the product by the lead; the quotient is the cotangent of the tooth angle of the worm.

Rules for Dimensioning the Worm-wheel. — The dimensions of the worm-wheel, named in the diagram shown in Fig. 3, are derived from the number of teeth determined upon for it, and the dimensions of the worm with which it is to mesh. The following rules may be used:

Rule 10. To find the pitch diameter of the worm-wheel, multiply the number of teeth in the wheel by the linear pitch of the worm, and divide the product by 3.1416.

Rule 11. To find the throat diameter of the worm-wheel, add twice the addendum of the worm tooth to the pitch diameter of the worm wheel.

Rule 12. To find the radius of curvature of the worm-wheel throat, subtract twice the addendum of the worm tooth from half the outside diameter of the worm.

The face angle of the wheel is arbitrarily selected; 60 degrees is a good angle, but it may be made as high as 80 or even 90 degrees, though there is little advantage in carrying the gear around

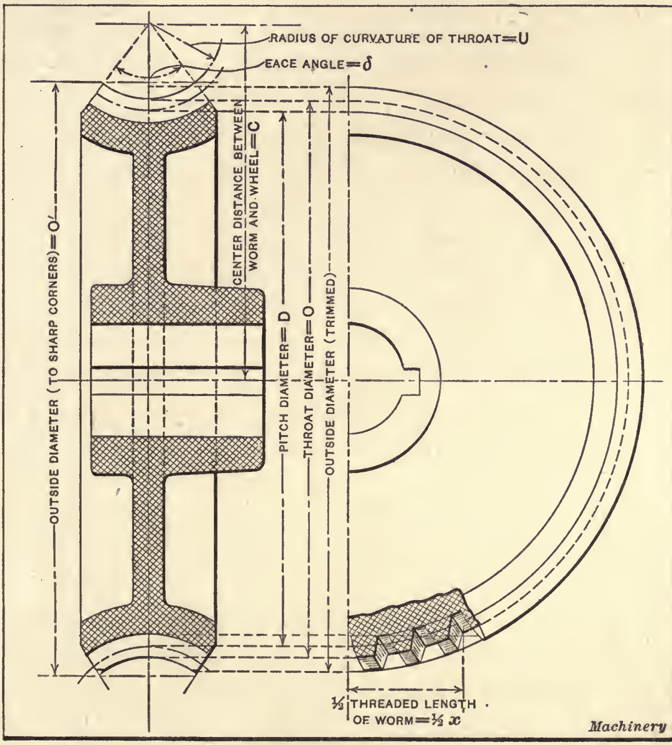


Fig. 3. Nomenclature used for Worm-gear Dimensions

so great a portion of the circumference of the worm, especially in steep pitches.

Rule 13. To find the diameter of the worm-wheel to sharp corners, multiply the radius of curvature of the throat by the cosine of half the face angle, subtract this quantity from the radius of curvature of throat, multiply the remainder by 2, and add the product to the throat diameter of the worm-wheel.

If the sharp corners are flattened a trifle at the tops, as shown in Figs. 3 and 5, this dimension need not be figured, "trimmed diameter" being easily scaled from an accurate drawing of the gear.

There is a simple rule which, rightly understood, may be used for obtaining the velocity ratio of a pair of gears of any form, whether spur, spiral, bevel or worm. The number of teeth of the driven gear, divided by the number of teeth of the driver, will give the velocity ratio. For worm gearing this rule takes the following form:

Rule 14. To find the velocity ratio of a worm and worm-wheel, divide the number of teeth in the wheel by the number of threads in the worm.

Be sure that the proper meaning is attached to the phrase "number of threads" as explained before under Rule 1. The revolutions per minute of the worm, divided by the velocity ratio, gives the revolutions per minute of the worm-wheel.

Rule 15. To find the distance between the center of the worm-wheel and the center of the worm, add together the pitch diameter of the worm and the pitch diameter of the worm-wheel, and divide the sum by 2.

Rule 16. To find the pitch diameter of the worm, subtract the pitch diameter of the worm-wheel from twice the center distance.

The worm should be long enough to allow the wheel to act on it as far as it will. The length of the worm required for this may be scaled from a carefully-made drawing, or it may be calculated by the following rule:

Rule 17. To find the minimum length of worm for complete action with the worm-wheel, subtract four times the addendum of the worm thread from the throat diameter of the wheel, square the remainder, and subtract the result from the square of the throat diameter of the wheel. The square root of the result is the minimum length of worm advisable.

The length of the worm should ordinarily be longer than the dimension thus found. Hobs, particularly, should be long enough for the largest wheels they are ever likely to be called upon to cut.

Departures from the Foregoing Rules. — The throat diameter of the wheel and the center distance may have to be altered in some cases from the figures given by the preceding rules. If worm-wheels with small numbers of teeth are made to the dimensions given, it will be found that the flanks of the teeth will be partly cut away by the tops of the hob teeth, so that the full bearing area is not available. The matter becomes serious when there are less than 25 or 30 teeth in the worm-wheel. One method of avoiding this under-cutting is to increase the throat diameter of the wheel blank in accordance with the following rule: To obtain the throat diameter, multiply the pitch diameter of the wheel by 0.937 and add to the product 4 times the addendum of the worm-wheel tooth. This diameter can also be obtained as follows: Multiply the product of the circular pitch and number of teeth in the worm-wheel by 0.298; then add 1.273 times the circular pitch. If it is necessary to keep the original center-to-center distance, the outside diameter of the worm must be reduced the same amount that the throat diameter is increased. When turning blanks, it is the general practice to simply reduce the central part of the throat to the required diameter, the remainder being left somewhat over size so that the tops of the teeth will be finished to the proper radius by the hob.

On the other hand, some designers claim to get better results in efficiency and durability by making the throat diameter of the worm-wheel *smaller* than standard, where it is possible to do so without too much under-cutting. Under no conditions, however, should the throat diameter ever be made so small as to produce more interference than is met with in a standard 25-tooth worm-wheel.

Two Applications of Worm Gearing. — Worm-wheels are used for two purposes. They may be employed to transmit power where it is desired to make use of the smoothness of action which they give, and the great reduction in velocity of which they are capable; instances of this application of worm gearing are found in the spindle drives of gear cutters and other machine tools. They are also used where a great increase in the effective power is required; in this case advantage is generally taken of the possibility of making the gearing self-locking. Such service is usually intermittent or occasional, and the matter of waste of power is not of so great importance as in the first case. Examples of this application are to be found in the adjustments of a great many machine tools, in training and elevating gearing for ordnance, etc. Calculations for the general design of this class of gearing will be treated separately in following chapters. In the case of elevator gearing and worm feeds for machinery, the functions of the gearing are, in a measure, a combination of those in the two applications.

Examples of Worm Gearing Figured from the Rules. — To show how the rules given above may be applied, we will work out two examples. The first of these is for a light machine tool spindle drive, in which power is to be transmitted continuously. It is determined that the velocity ratio shall be 8 to 1, and that the proper linear pitch to give the strength and durability required shall be about $\frac{3}{4}$ inch; the center distance is required to be 5 inches exactly. This case comes under the first of the two applications just described.

Assume, for instance, 32 teeth in the wheel, and a quadruple-thread worm. We will figure the gearing with these assumptions, and see if it appears to have practical dimensions.

The pitch diameter of the worm-wheel by Rule 10 is found to be

$$\frac{32 \times \frac{3}{4}}{3.1416} = 7.6394 \text{ inches.}$$

The pitch diameter of the worm by Rule 16 is found to be

$$(2 \times 5) - 7.6394 = 2.3606 \text{ inches.}$$

The addendum of the worm thread by Rule 5 is found to be

$$0.3183 \times \tfrac{3}{4} = 0.2387 \text{ inch.}$$

The outside diameter of the worm by Rule 6 is found to be

$$2.3606 + (2 \times 0.2387) = 2.8380 \text{ inches.}$$

For transmission gearing the angle of inclination of the worm thread should not be less than 18 degrees or thereabouts, and the nearer 30 or even 40 degrees it is, the more efficient will it be. From Rule 1 we find the lead to be $4 \times \frac{3}{4} = 3$ inches.

The helix angle of the worm thread is found from Rule 9 to be 2.3606 × 3.1416 ÷ 3 = 2.4722 = cot 22 degrees, approximately. This angle will give fairly satisfactory results. The calculations are not carried any further with this problem, whose other dimensions are determined from those just found. In the following case, however, all the calculations are made.

For a second problem let it be required to design worm-feed gearing for a machine to utilize a hob already in stock. This

Dimensions of Worm-thread Parts

Number of Threads per Inch	Circular or Linear Pitch, Inches	Circ. or Lin. Pitch, Decimal Equivalents	Height of Tooth above Pitch Line	Depth of Space below Pitch Line	Whole Depth of Tooth	Thickness of Tooth on Pitch Line	Width of Thread Tool at End	Width of Thread at Top
½	2	2.0000	0.6366	0.7366	1.3732	1.0000	0.6200	0.6708
4/7	1¾	1.7500	0.5570	0.6445	1.2015	0.8750	0.5425	0.5869
⅔	1½	1.5000	0.4775	0.5524	1.0299	0.7500	0.4650	0.5031
⅘	1¼	1.2500	0.3979	0.4603	0.8582	0.6250	0.3875	0.4192
1	1	1.0000	0.3183	0.3683	0.6866	0.5000	0.3100	0.3354
1⅓	¾	0.7500	0.2387	0.2762	0.5149	0.3750	0.2325	0.2515
1½	⅔	0.6667	0.2122	0.2455	0.4577	0.3333	0.2066	0.2236
2	½	0.5000	0.1592	0.1841	0.3433	0.2500	0.1550	0.1677
2½	⅖	0.4000	0.1273	0.1473	0.2746	0.2000	0.1240	0.1341
3	⅓	0.3333	0.1061	0.1228	0.2289	0.1667	0.1033	0.1118
3½	2/7	0.2857	0.0909	0.1053	0.1962	0.1429	0.0886	0.0958
4	¼	0.2500	0.0796	0.0920	0.1716	0.1250	0.0775	0.0838
4½	2/9	0.2222	0.0707	0.0819	0.1526	0.1111	0.0689	0.0745
5	⅕	0.2000	0.0637	0.0736	0.1373	0.1000	0.0620	0.0670
6	⅙	0.1667	0.0531	0.0613	0.1144	0.0833	0.0516	0.0559
7	1/7	0.1429	0.0455	0.0526	0.0981	0.0714	0.0443	0.0479
8	⅛	0.1250	0.0398	0.0460	0.0858	0.0625	0.0387	0.0419
9	1/9	0.1111	0.0354	0.0409	0.0763	0.0556	0.0344	0.0373
10	1/10	0.1000	0.0318	0.0369	0.0687	0.0500	0.0310	0.0335
12	1/12	0.0833	0.0265	0.0307	0.0572	0.0416	0.0258	0.0279
14	1/14	0.0714	0.0227	0.0263	0.0490	0.0357	0.0221	0.0239
16	1/16	0.0625	0.0199	0.0230	0.0429	0.0312	0.0194	0.0209
18	1/18	0.0556	0.0177	0.0205	0.0382	0.0278	0.0172	0.0186

hob is double-threaded, $\frac{1}{2}$ inch linear pitch, and $2\frac{1}{2}$ inches diameter. The center distance of the gearing is immaterial, but it is decided that the worm-wheel ought to have about 45 teeth to bring the ratio right. The only calculations made are those necessary for the dimensions which would appear on the shop drawing.

To find the lead, use Rule 1: 0.5 × 2 = 1.0 inch.

To find the whole depth of the worm tooth, use Rule 3: $0.5 \times 0.6866 = 0.3433$ inch.

To find the addendum, use Rule 5: $0.5 \times 0.3183 = 0.15915$ inch.

To find the pitch diameter of the worm, use Rule 7: $2.5 - 2 \times 0.15915 = 2.1817$ inches.

To find the bottom diameter of the worm, use Rule 8: $2.5 - 2 \times 0.3433 = 1.8134$ inch.

To find the gashing angle of the worm-wheel, use Rule 9: $2.18 \times 3.14 \div 1 = 6.845 = \cot 8$ degrees, 20 minutes, about.

To find the pitch diameter of the worm-wheel, use Rule 10: $45 \times 0.5 \div 3.1416 = 7.1620$ inches.

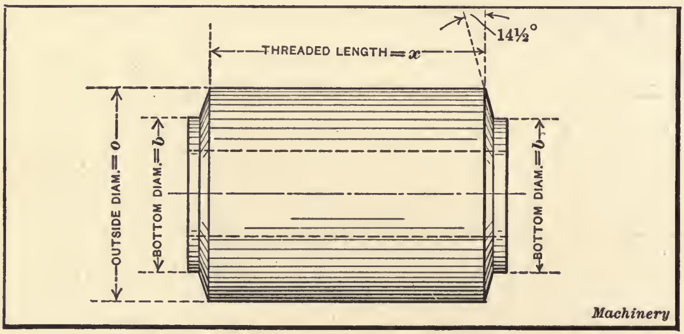


Fig. 4. Shape of Blank for Worm

To find the throat diameter of the worm-wheel, use Rule 11: $7.1620 + 2 \times 0.15915 = 7.4803$ inches.

To find the radius of the throat of the worm-wheel, use Rule 12: $(2.5 \div 2) - (2 \times 0.15915) = 0.9317$ inch.

The angle of face may be arbitrarily set at, say, 75 degrees, in this case. The "trimmed diameter" is scaled from an accurate drawing and proves to be 7.75 inches.

To find the distance between centers of the worm and wheel, use Rule 15: $(2.1817 + 7.1620) \div 2 = 4.6718$ inches.

To find the minimum length of threaded portion of the worm, use Rule 17: $7.4803 - 4 \times 0.15915 = 6.8437$.

$$\sqrt{7.4803^2 - 6.8437^2} = 3 \text{ inches, approximately.}$$

It will be noted that the ends of the threads in Fig. 2 are trimmed at an angle instead of being cut square down, as in Fig. 1. This gives a more finished look to the worm. It is easily done by applying the sides of the thread tool to the blank just before threading, or it may be done as a separate operation in preparing the blank, which will in either case have the appearance shown in Fig. 4. The small diameters at either end of the blank in Fig. 4 should, in any event, be turned exactly to the bottom diameter shown in Fig. 2, and obtained by Rule 8. This is of great assistance to the man who threads the worm, as he knows that the threads are sized properly as soon as he has cut down to this diameter with the end of his thread tool. This always requires, of course, that the thread tool is accurately made.

Formulas for the Design of Worm Gearing. — For the convenience of those who prefer to have their rules compressed into formulas, they are so arranged in the table on the following pages. The reference letters used are as follows:

P = circular pitch of wheel and linear pitch of worm;
l = lead of worm;
n = number of teeth or threads in worm;
S = addendum, or height of worm tooth above pitch line;
d = pitch diameter of worm;
D = pitch diameter of worm-wheel;
o = outside diameter of worm;
O = throat diameter of worm-wheel;
O' = outside diameter of worm-wheel (to sharp corners);
b = bottom or root diameter of worm;
N = number of teeth in worm-wheel;
W = whole depth of worm tooth;
T = width of thread tool at end;
α = face angle of worm-wheel;
β = helix angle of worm and gashing angle of wheel;
U = radius of curvature of worm-wheel throat;
C = distance between centers;
x = threaded length of worm.

Rules and Formulas for Worm Gearing*

To Find	Rule	Formula
Linear Pitch.	Divide the lead by the number of threads. — It is understood that by the number of threads is meant, not number of threads per inch, but the number of threads in the whole worm — one, if it is single-threaded, four, if it is quadruple-threaded, etc.	$P = \frac{l}{n}$
Addendum of Worm Tooth.	Multiply the linear pitch by 0.3183.	$S = 0.3183\,P$
Pitch Diameter of Worm.	Subtract twice the addendum from the outside diameter.	$d = o - 2\,S$
Pitch Diameter of Worm-wheel.	Multiply the number of teeth in the wheel by the linear pitch of the worm, and divide the product by 3.1416.	$D = \frac{NP}{3.1416}$
Center Distance between Worm and Gear.	Add together the pitch diameter of the worm and the pitch diameter of the worm-wheel, and divide the sum by 2.	$C = \frac{D + d}{2}$
Whole Depth of Worm Tooth.	Multiply the linear pitch by 0.6866.	$W = 0.6866\,P$
Bottom Diameter of Worm.	Subtract twice the whole depth of tooth from the outside diameter.	$b = o - 2\,W$
Helix Angle of Worm.	Multiply the pitch diameter of the worm by 3.1416, and divide the product by the lead; the quotient is the cotangent of the tooth angle of the worm.	$\cot \beta = \frac{3.1416\,d}{l}$
Width of Thread Tool at End.	Multiply the linear pitch by 0.31.	$T = 0.31\,P$
Throat Diameter of Worm-wheel.	Add twice the addendum of the worm tooth to the pitch diameter of the worm-wheel.	$O = D + 2\,S$
Radius of Worm-wheel Throat.	Subtract twice the addendum of the worm tooth from half the outside diameter of the worm.	$U = \frac{o}{2} - 2\,S$

* From MACHINERY'S HANDBOOK.

Rules and Formulas for Worm Gearing — (*Continued*)

To Find	Rule	Formula
Diameter of Worm-wheel to Sharp Corners.	Multiply the radius of curvature of the worm-wheel throat by the cosine of half the face angle, subtract this quantity from the radius of curvature, multiply the remainder by 2, and add the product to the throat diameter of the worm-wheel.	$O' = 2\left(U - U \times \cos\frac{\alpha}{2}\right) + O$
Minimum Length of Worm for Complete Action.	Subtract four times the addendum of the worm thread from the throat diameter of the wheel, square the remainder, and subtract the result from the square of the throat diameter of the wheel. The square root of the result is the minimum length of worm advisable.	$x = \sqrt{O^2 - (O - 4S)^2}$
Outside Diameter of Worm.	Add together the pitch diameter and twice the addendum.	$o = d + 2S.$
Pitch Diameter of Worm.	Subtract the pitch diameter of the worm-wheel from twice the center distance.	$d = 2C - D$

Worms with Large Helix Angle. — When worms have a large helix angle (15 degrees or more) the dimensions of the thread should be measured at right angles to the helix. In such cases, the following changes should be made in the formulas in the table and in the corresponding rules. Let:

$$P_n = \text{normal circular pitch} = P \times \cos\beta.$$

Then the formulas giving addendum, whole depth of worm tooth and width of thread tool at end will be written as follows:

$$S = 0.3183\, P_n; \qquad W = 0.6866\, P_n; \qquad T = 0.31\, P_n.$$

When these changes are made all the other formulas will give correct results when used in their original form.

Table for Calculating the Outside Diameter of Worm-wheels. — The regular formula for calculating the outside diameter (to sharp corners) of a worm-wheel is:

$$O' = 2\left(U - U\cos\frac{\alpha}{2}\right) + O,$$

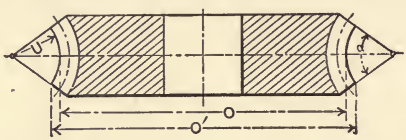
in which O' = the outside diameter of worm-wheel to sharp corners; U = the radius of the curvature of worm-wheel throat; α = face angle of worm-wheel; O = throat diameter of worm-wheel.

By writing this formula in the form:

$$O' = 2U\left(1 - \cos\frac{\alpha}{2}\right) + O$$

it will be seen that the expression within the parentheses can be tabulated for various face angles, and such a table is given here-

Table of Factors C Used in Worm-gear Formula



Angle α, Degrees	Factor C	Angle α, Degrees	Factor C	Angle α, Degrees	Factor C	Angle α, Degrees	Factor C
30	0.034	46	0.080	62	0.143	78	0.223
31	0.036	47	0.083	63	0.147	79	0.228
32	0.039	48	0.086	64	0.152	80	0.234
33	0.041	49	0.090	65	0.157	81	0.240
34	0.044	50	0.094	66	0.161	82	0.245
35	0.046	51	0.097	67	0.166	83	0.251
36	0.049	52	0.101	68	0.171	84	0.257
37	0.052	53	0.105	69	0.176	85	0.263
38	0.054	54	0.109	70	0.181	86	0.269
39	0.057	55	0.113	71	0.186	87	0.275
40	0.060	56	0.117	72	0.191	88	0.281
41	0.063	57	0.121	73	0.196	89	0.287
42	0.066	58	0.125	74	0.201	90	0.293
43	0.070	59	0.130	75	0.207	......	......
44	0.073	60	0.134	76	0.212	......	......
45	0.076	61	0.138	77	0.217	......	......

with. By using this table and calling the values found in the table for various angles C, the formula takes the simple form:

$$O' = 2U \times C + O,$$

in which C can be found in the table for any angle from 30 to 90 degrees.

Model Worm-gear Drawing. — A model drawing of a worm-wheel and worm, properly dimensioned, is shown in Fig. 5.

This drawing follows, in general, the model drawings shown by Mr. Burlingame in the August, 1906, issue of MACHINERY, taken from the drafting-room practice of the Brown & Sharpe Mfg. Co. In cases where the worm-wheel is to be gashed on the milling machine before hobbing, the angle at which the cutter is set should also be given. This is the same as the angle of worm

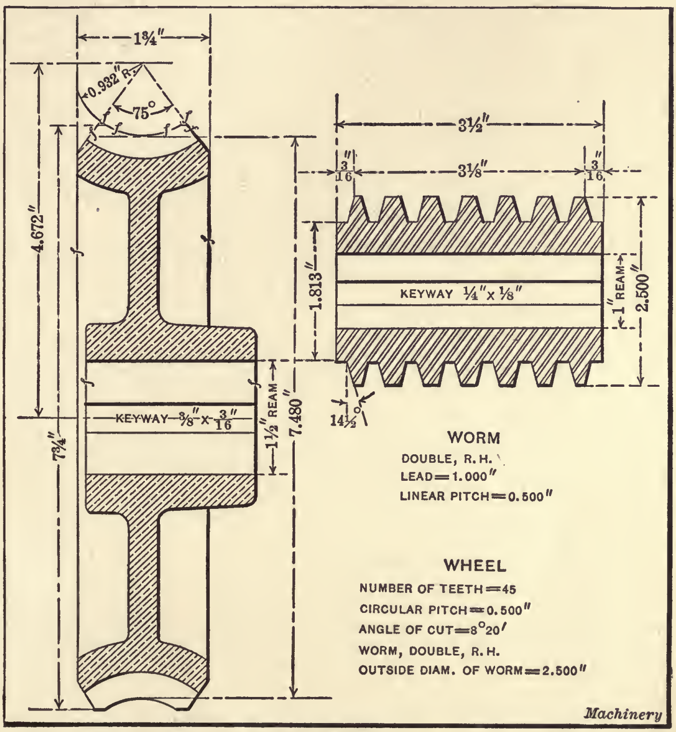


Fig. 5. Model Drawing of Worm and Worm-wheel

tooth found by Rule 9. In cases where the wheel is to be hobbed directly from the solid by a positively geared hobbing machine, this information is not needed. It might be added that it is impracticable with worm-wheels having less than 16 or 18 teeth to gash the wheel, and then hob it when running freely on centers, if the throat diameter has been determined by Rule 11.

CHAPTER VII

ALLOWABLE LOAD AND EFFICIENCY OF WORM GEARING

WHEN called upon to design a set of worm gearing for a certain drive, or select one from the catalogue of a manufacturer, the designer will find very little definite information in the ordinary textbooks on machine design concerning the allowable load, the allowable speed and the efficiency which may be expected — the very points which are of vital interest to him. The following paragraphs discuss these subjects.

Relation of Load to Effort. — In the following formulas let

P = pressure of the worm-wheel on the worm parallel to the worm-shaft;

F = force which must be applied at the pitch radius of the worm at right angles to the worm-shaft to overcome P;

α = angle of thread with a line at right angles to the axis of the worm;

f = coefficient of friction;

l = lead of worm thread;

d = pitch diameter of worm.

The normal pressure between worm and wheel then equals $F \times \sin\alpha + P \times \cos\alpha$, and the friction $f\,(F \times \sin\alpha + P \times \cos\alpha)$.

Now, if the worm is revolved once, we obtain the following relation between F and P:

$$F \times \pi d = Pl + f\,(F \times \sin\alpha + P \times \cos\alpha)\,\frac{\pi d}{\cos\alpha}$$

As $\dfrac{l}{\pi d} = \tan\alpha$, the formula above may be written:

$$F = P \times \frac{f + \tan\alpha}{1 - f\tan\alpha} \qquad (1)$$

This relation, giving the force F which must be applied at the pitch radius of the worm to overcome the load P at the pitch radius of the worm-gear, is often required by the designer.

Efficiency. — If there were no friction, or if f equalled 0, we would have:

$$F_1 = P \tan \alpha.$$

The efficiency of the worm gearing, is, therefore:

$$E = \frac{F_1}{F} = \frac{\tan \alpha (1 - f \tan \alpha)}{f + \tan \alpha} \qquad (2)$$

Equations (1) and (2) are, strictly speaking, only correct for worm threads with vertical sides, but the sloping thread side commonly used affects the result but little.

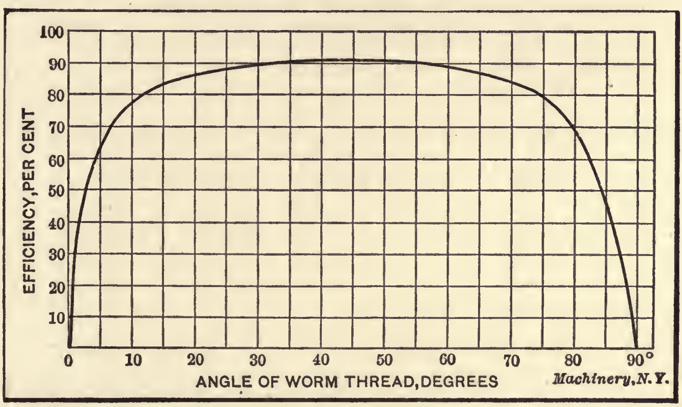


Fig. 1. Relation between Worm Thread Angle and Efficiency

To demonstrate the influence of the thread angle on the efficiency, the curve represented by Equation (2) with α and E as variables, and for a certain assumed value of f, has been plotted in Fig. 1. This curve is reproduced from "Worm and Spiral Gearing," by F. A. Halsey. It shows that the efficiency increases very rapidly with the thread angle for small angles, while for angles near the maximum efficiency, there is very little drop for a wide range of angles. It is, therefore, essential, for high efficiency, not to use thread angles that are too small. The value of f is found by experiments to vary with the speed of the rubbing surfaces. (See Transactions of the American Society of Mechanical Engineers, Vol. 7, page 273.) For values of f for various speeds see table, "Safe Load on Worm-gear Teeth."

Allowable Load. — It would seem reasonable to assume the allowable pressure on gear teeth under otherwise equal conditions to be expressed by

$$P = Cpb \tag{3}$$

where p = pitch, b = width of gear teeth, and C = a constant for the given speed.

For very slow speed, where there is no danger of overheating, and where the only questions to be taken into account are the strength and the resistance to abrasion, the above equation is

Relation Between Velocity at Pitch Line, Angle of Thread and Efficiency

Velocity at Pitch Line, Feet per Minute	Angle of Thread, Degrees					
	5	10	20	30	40	45
	Efficiency, Per Cent					
5	40	56	69	76	79	80
10	47	62	74	79	82	82
20	52	67	78	83	85	86
30	56	71	81	85	87	87
40	60	74	83	87	88	88
50	63	76	85	88	89	89
75	67	80	87	90	90	90
100	70	82	88	91	91	91
150	74	84	90	92	92	92
200	76	85	91	92	92	92

obviously correct if we assume some standard ratio of worm diameter to wheel face. A pitch diameter of worm equal to 1.5 times the face of the wheel (corresponding to a face angle of 76 degrees) is about right.

For higher speeds the amount of frictional work transformed into heat may cause an excessive rise in temperature before the worm, or casing surrounding the worm, is able to carry off the same amount of heat as is developed, and the limiting load will be determined thereby. The ability of the worm or casing to carry off heat is proportional to the surface, which, again, is approximately proportional to the product pb of the pitch and width of the gear teeth; hence, Equation (3) is approximately correct in this case also. This equation is given in "Des Ingenieurs Taschenbuch" (Hütte).

German Experiments to Determine Speed Factor. — The value of the factor C varies with the speed and must be determined by experiment. The most complete experiments to this effect are those of C. Bach and E. Roser, published in *Zeitschrift des Vereines Deutscher Ingenieure*, Feb. 14, 1903. These experiments were made with a three-threaded steel worm, not hardened; 76.6 millimeters (3 inches) pitch diameter; 25.4 millimeters (1 inch) pitch; 17 degrees 34 minutes, thread angle; 148 millimeters ($5\frac{13}{16}$ inches) long. The worm-wheel was of bronze, 242.6 millimeters ($9\frac{9}{16}$ inches) pitch diameter, with

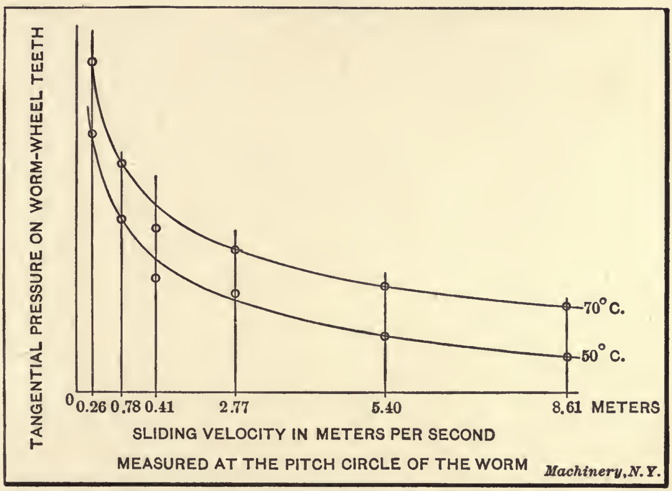


Fig. 2. Relation between Tangential Pressure and Velocity

milled teeth, 78 millimeters ($3\frac{1}{16}$ inches) wide measured on the arc, 30 teeth, speed ratio 1 to 10, with ball bearings for worm shaft, and oil bath of extremely viscous oil.

In the experiments the load on the teeth varied from 111 kilograms (244 pounds) to 1257 kilograms (2765 pounds) and the speed varied from 2185 R.P.M. to 64 R.P.M. The temperature of the oil bath and that of the surrounding air was observed until the difference reached a constant value. Corresponding values of load and speed for constant temperature difference were ascertained and an attempt to express the relation by an equation gave the following rather lengthy expression:

$$P = Cpb = [a\ (t_o - t_e) + d]\ pb \qquad (4)$$

in which

$$a = \frac{0.0669}{V} + 0.4192;$$

$$d = \frac{109.1}{V + 2.75} - 24.92;$$

t_o = temperature of oil in degrees C.;

t_e = temperature of air in degrees C.;

V = sliding velocity at pitch line in meters per second.

Safe Load on Worm-gear Teeth

Load per unit of the product (pitch × width of tooth), for 90-degree F. temperature difference between oil and surrounding air. More than 1000 pounds per unit of product (pitch × width of tooth) should not be allowed under ordinary circumstances. Cut bronze-gear, cut steel-worm.

Velocity in Feet per Minute	Load in Pounds per Unit of (Pitch × Width of Tooth)	Coefficient of Friction	Velocity in Feet per Minute	Load in Pounds per Unit of (Pitch × Width of Tooth)	Coefficient of Friction
5	1000	0.146	200	403	....
10	1000	0.116	250	371	....
20	956	0.090	300	341	....
30	790	....	350	315	....
40	703	0.070	400	292	....
50	646	....	500	257	....
75	564	....	600	228	....
80	554	0.054	700	206	....
100	514	....	800	185	....
125	477	....	900	167	....
150	448	....	1000	151	....
175	424	....	1200	128	....

The curves represented by Equation (4) are distinctly hyperbolic in character. Two of these curves have been plotted in Fig. 2, one for a temperature difference of 50 degrees C. and one for 70 degrees C.

The accompanying table shows the loads for various speeds, calculated from Equation (4) and transformed into English units, for a difference in temperature of 90 degrees F. (50 degrees C.). In the same table are given coefficients of friction as deduced by Unwin from Lewis's experiments. This table of load used with discretion, and with due consideration for the various individual conditions associated with the drive in contemplation, may be made the basis for worm-gear design in average cases and where a temperature rise of 90 degrees F. (50 degrees C.) is allowable. The loads given are for continuous service, and as it will take several minutes, perhaps hours, before the constant temperature is reached, a higher load will be justified for intermittent service, where the oil has time to cool down. It should be kept in mind that the danger of abrasion will, of course, depend on the temperature of the oil, and not on the temperature difference; if, therefore, the gearing is installed in a place where the surrounding temperature is kept low, the temperature difference can be correspondingly increased and *vice versa.* The danger of abrasion will also, to a large extent, depend on the character of the lubricant, in that a very viscous oil will offer greater resistance to the squeezing out of the oil film between the rubbing surfaces than the less viscous.

Practical Points in the Design of Worm and Gear. — It should be remembered also that a gear with many teeth gives a better contact with the worm both on account of the flatter curve of the engaging segment and the larger average radii of curvature of its teeth. This has particular reference to the heavy loads at slow speed, where the question of temperature does not enter.

The angle of thread (the helix angle) does not appear in the formula given, as it has no direct bearing on the question of allowable load and speed of rubbing surfaces. As previously mentioned, the angle of thread has, however, a direct influence on the efficiency of the gearing. Given, for instance, two worms of the same diameter, one having a thread angle twice as great as the other, carrying the same load on the gear teeth and running at the same speed, there is no reason at all why one should be more successful than the other as far as wearing qualities are concerned, but it must be remembered that the first one is transmitting twice the horsepower of the other, and will obviously give much better efficiency.

With the allowable load decreasing as the speed increases, as provided for by the formula and table given, a speed of rubbing surfaces as high as 1000 feet per minute, or even higher, can undoubtedly be used with success for cut gearing, which also has been demonstrated repeatedly in practice. In the tests by Bach and Roser, the speed was carried as high as 8.76 meters per second (1724 feet per minute), with a load of 370 kilograms (814 pounds) and a temperature difference of 80.5 degrees C. (126.9 degrees F.) with no apparent cutting. The loads given represent tangential loads at right angles to the worm-gear shaft. The actual pressure between the rubbing surfaces will be more, and will increase with the angle of thread, but the increase for gears in common use (less than 20-degree thread angle) is not very great.

Concerning the coefficient of friction f, this has not been deduced for higher speeds than 80 feet per minute, but it will be seen that there is a general tendency for the value of f to decrease as the speed increases.

Except for hand-operated gearing, or for machinery which is only operated occasionally and for a very short time, the worm and gear should be enclosed in an oil casing and the worm always placed below the gear to insure the submersion of the rubbing surfaces in oil. Except in the cases mentioned, machine-cut worms and wheels should always be used. Hardened steel worms working with bronze wheels have proved to give good satisfaction, because this combination wears longer than cast iron or steel and cast iron.

Self-locking Worm Gearing. — A set of worm gearing will be self-locking when the thread angle is equal to, or smaller than, the angle of friction. From Equation (2) we obtain, by making $f = \tan\alpha$, the efficiency of worm gearing having a thread angle just small enough to be self-locking, as follows:

$$E_1 = \frac{\tan\alpha\,(1 - \tan^2\alpha)}{2\tan\alpha} = \tfrac{1}{2}\,(1 - \tan^2\alpha) \qquad (5)$$

Equation (5) gives a maximum of E_1 for $\tan\alpha = 0$ or $\alpha = 0$, and this value is E_1 max. $= \frac{1}{2}$.

From this it will be seen that it is impossible to obtain an efficiency greater than 0.5 if the gears are to be self-locking in themselves. Of course, there will always be some friction in the worm-shaft bearings and other parts of the machinery which may prevent the pressure on the worm-gear from actually turning the machinery as a whole backwards, even if the angle of thread is larger than that of friction. This, in connection with the fact that the efficiency for backward movement is low, is probably the reason why many worm-gear drives, applied as self-locking, have angles of thread far in excess of the friction angle, and still seem to work satisfactorily.

On account of the variable coefficient of friction, the angle of thread which may safely be used for self-locking gears will also depend largely on the speed with which the machine is run backward, or, in other words, the speed with which the load is lowered or eased off by means of the worm-gear. If the machine is never run but one way, and the worm-gear applied as safety device to prevent backward movement in case of accident, then the load would have to start the worm shaft rotating, and a larger angle of thread could undoubtedly be used. The subject of self-locking worm gearing will be treated in greater detail in a following chapter.

An Example from Practice. — To indicate the use of the formulas and table in practical work, the following example has been prepared: Assume a set of worm gearing used for driving a package elevator with the worm-gear shaft running at a speed of 5 R.P.M. The required turning moment is 42,000 inch-pounds. It is desired to have the worm gearing self-locking to prevent the elevator from running backward in case the driving belt breaks or jumps off.

As the elevator must come to a stop before it can commence to run backward it is only necessary to have a thread angle equal to or smaller than the angle of friction for rest. Assuming the coefficient of friction to be at least 0.15 at rest, the thread angle α will be determined by $\tan\alpha = 0.15$, or $\alpha = 8\frac{1}{2}$ degrees. If the speed of the worm shaft is not dependent on other conditions, we have a choice between a single- and a double-threaded worm. A single-threaded worm of $1\frac{3}{4}$ inch pitch would have a diameter $= \frac{1\frac{3}{4}}{0.15\,\pi} = 3.71$ inches. This may not be enough to allow for a worm shaft of sufficient strength; besides it would give a very narrow face to the worm-gear. We, therefore, probably prefer to use a double-threaded worm, the pitch diameter of which will be $\frac{1\frac{3}{4} \times 2}{0.15\,\pi} = 7.42$ inches. The face of the worm-gear will then be

$$\tfrac{2}{3} \times 7.42 = 4.95 \text{ inches, or, say, } 5 \text{ inches.}$$

Assuming a worm-gear of 28 inches pitch diameter, $1\frac{3}{4}$ inch pitch, and 50 teeth, the worm shaft will be running $5 \times \frac{50}{2} =$ 125 R.P.M., which gives a speed of rubbing of $\frac{\pi \times 7.42 \times 125}{12 \times \cos 8\frac{1}{2} \text{ deg.}}$ = 247 feet per minute.

Referring to the table, "Safe Load on Worm-gear Teeth," we find for a speed of 250 feet per minute an allowable load of 371 pounds per unit of product (pitch × width of tooth). The total allowable load in this case will be $371 \times 1\frac{3}{4} \times 5 = 3246$ pounds. This load at 14 inches radius gives a turning moment of $14 \times 3246 = 45{,}444$ inch-pounds, while only 42,000 inch-pounds is required.

If the above machine were applied for lowering packages instead of elevating same, as previously assumed, the gearing would have to lock while running at a full speed of 247 feet per minute, at which speed we would not have a friction coefficient of more than 0.05, at the most, which would correspond to an angle of thread determined by $\tan \beta = 0.05$, or $\beta = 2$ degrees 50 minutes approximately, and the gear with a thread angle of $8\frac{1}{2}$ degrees could not be expected to lock.

To find the efficiency of the above gearing when running at full speed, assume a coefficient of friction of 0.05, and apply Formula (2) which gives

$$E = \frac{\tan \alpha\,(1 - f \tan \alpha)}{f + \tan \alpha} = \frac{0.15\,(1 - 0.05 \times 0.15)}{0.05 + 0.15} = 74 \text{ per cent.}$$

This is the efficiency of the worm gearing only and does not allow for the friction loss in the worm-gear shaft nor any frictional loss in the other parts of the machine.

To find the effort F which must be exerted at the pitch radius of the worm to turn the worm shaft with a load $= \frac{42{,}000}{14} = 3000$ pounds, at the worm-gear periphery, apply Formula (1) which gives (for $f = 0.15$ at starting):

$$F = P \times \frac{f + \tan\alpha}{1 - f\tan\alpha} = 3000 \times \frac{0.15 + 0.15}{1 - 0.15 \times 0.15} = 921 \text{ pounds.}$$

To this should be added the friction in the worm-shaft bearings reduced to the same radius.

Theoretical Efficiency of Worm Gearing — Oerlikon Experiments. — The following table gives the theoretical efficiency of worm gearing for a number of different coefficients of friction. Practical experiments carried out by the Oerlikon Company, Oerlikon by Zurich, Switzerland, agree closely with the results from theoretical calculations given in the table. These experiments indicate that the efficiency increases with the angle of

Table Giving Theoretical Efficiency of Worm Gearing

Coefficient of Friction	Angle of Inclination								
	5 deg.	10 deg.	15 deg.	20 deg.	25 deg.	30 deg.	35 deg.	40 deg.	45 deg.
0.01	89.7	94.5	96.1	97.0	97.4	97.7	97.9	98.0	98.0
0.02	81.3	89.5	92.6	94.1	95.0	95.5	95.9	96.0	96.1
0.03	74.3	85.0	89.2	91.4	92.7	93.4	93.9	94.1	94.2
0.04	68.4	80.9	86.1	88.8	90.4	91.4	92.0	92.2	92.3
0.05	63.4	77.2	83.1	86.3	88.2	89.4	90.1	90.4	90.5
0.06	59.0	73.8	80.4	84.0	86.1	87.5	88.2	88.6	88.7
0.07	55.2	70.7	77.8	81.7	84.1	85.6	86.4	86.9	86.9
0.08	51.9	67.8	75.4	79.6	82.2	83.8	84.7	85.2	85.2
0.09	48.9	65.2	73.1	77.6	80.3	82.0	83.0	83.5	83.5
0.10	46.3	62.7	70.9	75.6	78.5	80.3	81.4	81.9	81.8

inclination, up to a certain point. They also show that for larger angles of inclination than from 25 to 30 degrees the efficiency increases very little, especially if the coefficient of friction is small, and this fact is of importance in practice, because, for reasons of gear ratio and conditions of a constructive nature, an angle greater than 30 degrees cannot be employed. The coefficient of friction increases with the load and diminishes to a certain extent with the increase of speed. Besides the friction between the worm and the wheel teeth, there is also the friction of the spindle bearings and the ball bearings for taking the axial thrust. To obtain the best results, there must be very careful choice of dimensions of teeth, of the stress between them, and the angle of inclination. To show what can be done, the following are the results of a test with an Oerlikon worm-gear for a colliery winding engine: The motor gave 30 brake horsepower to 40 brake horsepower at 780 revolutions. The normal load was 25 brake horsepower, but at starting it could develop 40 brake horsepower. The worm-gear ratio was 13.6 to 1, the helicoidal bronze wheel having 68 teeth on a pitch circle of 7.283 inches, and the worm 5 threads. The power required at no load for the whole mechanism was 520 watts, corresponding to 2.8 per cent of the normal. The efficiency at one-third normal load gave 90 per cent, at full load 94½, and at 50 per cent overload 93 per cent. The efficiency of the *worm and wheel* alone is higher, and, knowing the no-load power, is calculated to be 97½ per cent. According to the table given, of theoretical efficiencies, this gives the coefficient of friction as 0.01. To obtain a reduction of 13.6 to 1 with spur gears would have necessitated two pinions and two wheels with their spindles and bearings, and if the bearing friction was taken into consideration, the efficiency of such gearing would certainly not have reached the above-mentioned figure of 94½ per cent at full load. These figures, of course, seem very high for the efficiency of worm gearing. They were published in MACHINERY, December, 1903, having been obtained from a reliable source, and were never challenged. They have also been published in several editions of MACHINERY'S Reference Book No. 1, "Worm Gearing," without adverse criticism.

Worm and Helical Gears as Applied to Automobile Rear-axle Drives. — European practice extending over a period of fifteen years has given ample evidence of the eminent success of the worm and helical type of gearing, and in a paper read before the Society of Automobile Engineers, Mr. F. Burgess, the well-known gear expert, stated that he felt confident in saying that in the near future a large percentage of the cars in the United States will be equipped with this drive. The principal reason for the adoption of the helical form of tooth appears to be its peculiar quality of silence, regardless of speed or load. With the best methods of design and assembly, great durability, strength and efficiency are obtained.

The successful worm-gear should embody the following qualifications:

1. Cheapness of construction.
2. Strength for resisting shocks.
3. Hardened and smooth surfaces for durability.
4. Material of a suitable composition to reduce friction.
5. Simplicity of construction and mounting.
6. Perfect bearing conditions.
7. Noiselessness at any speed or load.
8. Reversibility.
9. Lightness in weight.
10. High efficiency in power transmission.

Granting that there is some argument against the worm in regard to trucks as to the dead axle proposition, this could be overcome by using a worm-gear on each end of the axle, the

Results of Efficiency Tests on Ordinary Type Worm-gear for Automobile Rear-axle Drive for Electric Vehicles and Light-power Cars

Number of Test	Temperature of Worm-gear, Degrees F.	Twist of Shaft, Degrees	R.P.M. of Worm	R.P.M. of Worm-gear	Input, Transmission Dynamometer, Horse-power	Output, Brake Horse-power	Efficiency, Per Cent
1	74	1¼	1393	143	1.64	1.01	61.6
2	82	2	1423	146	2.65	2.11	79.6
3	86	2¾	1416	145	3.41	3.11	91.3
4	86	3 9/16	1416	145	4.46	4.15	93
5	90	4 7/16	1370	140.5	5.48	5.03	92
6	94	5 7/16	1389	142.5	6.72	6.12	91.2

Worm-gear: Phosphor-bronze, 39 teeth.

Worm: Casehardened steel worm, solid on shaft, quadruple thread.

same as sprocket wheels, having a double worm-gear drive in place of the cumbersome chain drive. If at first this is slightly more expensive than the chain and sprocket drive, less repairs will more than make up the difference. Care should be taken to have accurate bearings, both radial and end-thrust.

Considerable discussion has arisen in regard to the relative merits of the straight and Hindley types of worm gearing. Both can be used successfully, although each has its own advantages and disadvantages. For most purposes, particularly where considerable power is to be transmitted, the Hindley type has the advantage, but with ordinary machinery it is somewhat more difficult to obtain the same degree of accuracy as can be obtained in the case of the straight type.

From tests made there is no question but that there is a larger bearing surface on the Hindley type of worm than on the straight. Therefore this type of gearing will for the same pitch present a bearing of greater durability, and heat less than the straight type, particularly under heavy load. The straight type may have less trouble with end-thrust bearings. The worm can move in its position longitudinally with the worm axis and therefore does not require as close an adjustment of the end-thrust bearings. With first-class bearings the Hindley type has the advantage, as a smaller and lighter gear can be used, thus reducing the expense.

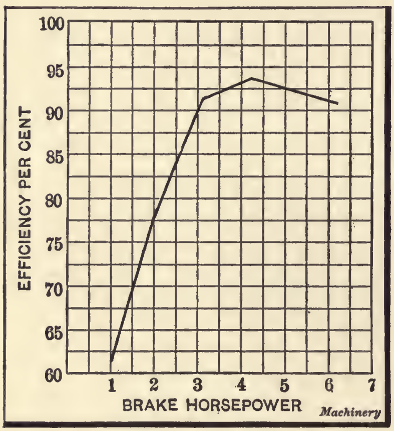


Fig. 3. Diagram showing Relation between Brake Horsepower and Efficiency, based on Results of Tests

Some efficiency tests on an ordinary type worm and worm-gear for automobile rear-axle drive, for electrical vehicles and light-power cars, were undertaken by Mr. Burgess. A transmission dynamometer, similar in some respects to the apparatus used at the Massachusetts Institute of Technology by Professor Riley, was constructed. The prony brake was adopted for an absorption dynamometer, and a long shaft of small diameter was arranged to obtain the torsion of the shaft in degrees by an electrical indicator apparatus for a transmission dynamometer. The results of the tests are given in the accompanying table.

The diagram, Fig. 3, gives a curve plotted from the results obtained in the tests and recorded in the table.

Worm Gearing Employed for Freight Elevators. — In general the worm should be made just as small as the circumstances will allow in order to increase the angle of thread and thereby the efficiency, while maintaining the same pitch and the same number of threads on the worm.

There are three factors which may determine the minimum size of worm that can be used, which are as follows: First, the diameter of the shaft on which the worm has to be keyed, if not made in one piece with this shaft, limits the size of the worm. Second, if the gear is to be self-locking the angle of the thread cannot be increased above a certain degree; with the pitch settled on, this will determine the diameter of the worm, provided it is single threaded. Third, if the face of the gear is determined, it is not desirable to go below a certain diameter of worm on account of the consequent large face angle.

Factors Determining the Load. — Concerning the load which can safely be carried on worm gearing, it is determined by one of three considerations, which are: the strength of the material, the danger of abrasion and the danger of overheating.

The first consideration seldom comes into play because a gear proportioned to prevent abrasion and excessive heating will generally have excessive strength. For very slow-running worms and for worms used intermittently with short runs and long intervals, the heating effect does not enter and the determining factor will be the danger of abrasion from too high a pressure per unit of contact surface. The contact between worm and worm-gear is mathematically a line, but the physical properties of the opposed surfaces and the lubricant between them expand this ideal line into an actual area, and as the radii of curvature increase directly with the pitch, it is natural to consider this surface as directly proportional to the product of pitch and face. The proper allowable load per unit must necessarily be determined by experience, and 1000 pounds to 1200 pounds seems to be about the safe limit of load per unit of $p \times f$ (pitch $\times$ face) considering that there ought to be here, as well as in all other designs, a certain margin or factor of safety, as we might say, to prevent having the machine put out of commission by an occasional overload or other accidental excessive pressure. If all the load, as is usual for spur gears, is considered to be taken by one tooth, the stresses produced in the material for these loads are about the safe stresses for cast iron.

Overheating. — The third consideration, the danger of overheating, is perhaps the most important, and the most trouble with worm gearing is from this source. When more heat is developed than is carried off from the gear housing, the temperature of the oil will increase, but with the higher temperature the oil becomes less viscous and its adhesion to the rubbing surfaces becomes less. The coefficient of friction increases with consequent more rapid increase in temperature. Thus the critical conditions are constantly augmented by one another until the oil film between the surfaces is squeezed out altogether and abrasion occurs. The only safe way to avoid this is, of course, to so design the gearing that the temperature is kept below a certain limit. For continuous service the proper loads may be based on Bach's and Roser's experiments referred to in the preceding sections of this chapter. It may be well here to call attention to the fact that the loss of heat from a body is approximately directly proportional to its surface, and consequently a large gear housing is at an advantage. The housing should have stuffing-boxes for the worm shaft, and be well filled with a viscous oil so that the heat created at the point of contact may be distributed quickly to the upper parts of the housing.

Special Application to Freight Elevators. — For intermittent duty, like that imposed on a freight elevator, the question of allowable load becomes more complicated. The load that can safely be carried on a gear for this class of work will depend entirely on the circumstances, and a value can only be arrived at if these are known, or after certain assumptions as to the maximum time of continuous service, time of intervals, etc., have been made. The total heat developed can then be compared to that for continuous service and a correspondingly higher load allowed.

Consider, for instance, a worm for driving a freight elevator with a load on a 24-inch drum of 4000 pounds, worm direct on motor shaft running at 850 R.P.M. If, in this instance, it is considered safe to assume that the maximum average load for a certain unit of time will never exceed 2000 pounds and the time required for loading and unloading the elevator is at least equal to the time of actual running, then the work performed by the worm gearing will be one-fourth of that for continuous service with full load, assuming the coefficient of friction the same for all loads. The heat developed will also be one-fourth of that developed with full load. A gearing designed for continuous service with 1000 pounds load on drum will therefore meet the requirements.

For a worm of $3\frac{3}{4}$ inches in diameter running at 850 R.P.M., we have a velocity of 824 feet per minute. For this velocity a load per unit of pitch times face of 180 pounds is allowable for a difference in temperature of 50 degrees F. The gear will have to have 108 teeth to give the necessary reduction to 50 feet per minute elevating. As this number of teeth is exceptionally large, we can expect a good contact with less danger of abrasion, and a higher temperature difference, say, 70 degrees, is warranted. The allowable load is approximately proportional to the temperature difference, and we can, therefore, allow $180 \times \frac{70}{50} = 252$ pounds per unit of pitch times face. On account of the large diameter of the worm-gear, the worm and its housings will be comparatively long with consequent large radiating surface, and a temperature difference of 70 degrees will probably not be reached at all.

A worm, 4 inches in diameter, 1 inch pitch, with gear 34.4 inches in diameter, $2\frac{3}{4}$ inches face, will then carry $252 \times 1 \times 2\frac{3}{4} = 693$ pounds, which corresponds to a load on the drum $= 693 \times \frac{34.4}{24} = 993$ pounds, or practically 1000 pounds. The intermittent load on gears will be $4000 \times \frac{24}{34.4} = 2791$ pounds, or 1015 pounds per $(p \times f)$, which in this case is within the limit.

Frequently Employed Objectionable Designs. — Many worm-gearing designs used on freight elevators employ too high a load on the gear teeth. In one case a worm-gear having 108 teeth, $\frac{3}{4}$ inch pitch, $2\frac{1}{2}$ inches face and a worm $5\frac{3}{16}$ inches in diameter, single threaded, was used. The worm was direct-connected to an electric motor running at 850 revolutions per minute. Difficulties were experienced with regard to the heating of the worm. The current required by the motor was also too great. The winding drum was 24 inches in diameter and the load on the drum 4000 pounds, which corresponds to 3720 pounds on the worm-gear teeth.

With the dimensions given, the angle of thread is 2 degrees 39 minutes, and the efficiency of the worm-gearing for a coefficient of friction equal to 0.05 would be 0.48 (see preceding sections of this chapter). If the diameter of the worm were reduced to $3\frac{3}{4}$ inches, the angle of the thread would be increased to 3 degrees 39 minutes and the efficiency to 0.58, an increase in efficiency of 21 per cent. It will be seen from this that a decrease in worm diameter not only reduces the speed of the rubbing surfaces, but also increases the efficiency.

A load on the gear teeth of 3720 pounds for a $\frac{3}{4}$-inch pitch, $2\frac{1}{2}$-inch face gear, corresponds to a load per unit of $p \times f$ equal to 1984 pounds. This is without question too heavy a load, even for intermittent service, and worm gearing with any such load, running at high speed, is likely to give trouble. Upon being advised that the gears, as described, were too small for the service required of them, the manufacturers of the gearing stated that they were building elevators in competition with other concerns and that a material increase in these gears would make it impossible for them to compete successfully. They also stated that they had sometimes operated a load of 6000 pounds with a 10-horsepower motor, but in one or two cases they had found it very difficult to start the elevator except by using a heavy current.

Now 6000 pounds at 50 feet per minute represents $\frac{6000 \times 50}{33,000}$ $= 9.1$ H.P. The efficiency of a worm-gear with an angle of thread 2 degrees 40 minutes was found above to be 0.48 for a coefficient of friction of 0.05. This is the efficiency of the worm gearing itself and does not allow for friction in gear or worm-shaft bearings, for end-thrust bearing, for bending of cables or friction in guides. When all this is taken into consideration the horsepower required for running conditions will be at least 22. The horsepower for starting will be still higher and a correspond-

Horsepower Transmitted by Worm Gearing, Single-threaded Worm
(Gear: Phosphor-bronze; Worm: Hardened Steel)

Horsepower to be Transmitted	Single-threaded Worm: Rev. per Minute of Worm	Single-threaded Worm: Linear Pitch of Worm, Inches	Single-threaded Worm: Max. Pitch Diam. of Worm, Inches
1	1000	1/2	7 3/4
	1200	7/16	7 1/2
	1500	7/16	7 1/4
	1800	3/8	6 3/4
	2400	3/8	6 1/2
	3000	5/16	6
1.5	300	3/4	8
	450	11/16	7 1/2
	600	5/8	7
	900	9/16	6 1/4
	1200	1/2	5 3/4
	1500	1/2	5 1/2
	1800	7/16	5 1/4
	2400	7/16	5
	3000	3/8	4 3/4
2	150	1	8
	300	13/16	7
	450	3/4	6 1/4
	600	11/16	5 3/4
	900	5/8	5 1/4
	1200	9/16	5
	1500	9/16	4 3/4
	1800	1/2	4 1/2
	2400	1/2	4 1/4
	3000	7/16	4

Horsepower to be Transmitted	Single-threaded Worm: Rev. per Minute of Worm	Single-threaded Worm: Linear Pitch of Worm, Inches	Single-threaded Worm: Max. Pitch Diam. of Worm, Inches
3	100	1 1/4	7
	150	1 1/8	6 1/2
	300	15/16	5 1/2
	450	7/8	5
	600	13/16	4 1/2
	900	3/4	4
	1200	11/16	3 3/4
	1500	5/8	3 1/2
	1800	5/8	3 3/8
	2400	9/16	3 1/4
	3000	9/16	3
4	100	1 7/16	6
	150	1 1/4	5 1/2
	300	1 1/16	4 1/2
	450	15/16	4
	600	7/8	3 3/4
	900	13/16	3 1/2
	1200	3/4	3 1/4
	1500	3/4	3
	1800	11/16	2 7/8
	2400	5/8	2 5/8
	3000	5/8	2 1/2
5	100	1 1/2	5
	150	1 3/8	4 1/2
	300	1 3/16	3 7/8

Horsepower to be Transmitted	Single-threaded Worm: Rev. per Minute of Worm	Single-threaded Worm: Linear Pitch of Worm, Inches	Single-threaded Worm: Max. Pitch Diam. of Worm, Inches
5	450	1 1/16	3 1/2
	600	15/16	3 1/4
	900	7/8	3
	1200	13/16	2 3/4
	1500	3/4	2 5/8
	1800	3/4	2 1/2
	2400	11/16	2 3/8
	3000	5/8	2 1/4
6	100	1 5/8	4 1/2
	150	1 1/2	4 1/4
	300	1 1/4	3 1/2
	450	1 1/8	3 1/4
	600	1 1/16	3
	900	15/16	2 5/8
	1200	7/8	2 1/2
	1500	13/16	2 3/8
	1800	13/16	2 1/4
	2400	3/4	2
8	100	1 3/4	3 3/4
	150	1 5/8	3 1/2
	300	1 3/8	2 7/8
	450	1 1/4	2 5/8
	600	1 1/8	2 1/2
	900	1 1/16	2 1/4
	1200	15/16	2 1/8

ing electric current consumption in the motor must necessarily result, which indeed must be called high for a 10-H.P. motor.

Lubricant for Worm-gears. — In the majority of installations the worms are cut from steel while the worm-wheels are of cast iron. A very satisfactory lubricant is composed of the following ingredients: Cylinder oil, 2 gallons; common flour, 1 pound; common salt, $\frac{1}{2}$ pound.

It will be found that the flour will make the oil heavy enough to stick while the salt will so glaze the worm and gear that they will run smoothly without any tendency to "score." In extreme cases, where the worm-gear runs hot, owing to continuous and fast running and to friction, it is sometimes advisable to add to the above $\frac{1}{2}$ pound of graphite. The lubricating and

Horsepower Transmitted by Worm Gearing, Double-threaded Worm

(Gear: Phosphor-bronze; Worm: Hardened Steel)

Horsepower to be Transmitted	Double-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
2	1200	1/2	8
	1500	7/16	7 3/4
3	500	11/16	8
	600	11/16	7 1/2
	750	5/8	7
	900	5/8	6 3/4
	1200	9/16	6 1/2
	1500	9/16	6
4	200	1	8
	300	7/8	7 1/2
	450	13/16	7
	600	3/4	6 1/2
	750	3/4	6
	900	11/16	5 3/4
	1200	5/8	5 1/4
	1500	5/8	5
5	150	1 3/16	7 3/4
	200	1 1/16	7 1/4
	300	15/16	6 1/2
	450	7/8	6
	600	13/16	5 1/2
	750	3/4	5 1/4
	900	3/4	5

Horsepower to be Transmitted	Double-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
5	1200	11/16	4 3/4
	1500	5/8	4 1/2
6	100	1 3/8	7 3/4
	150	1 1/4	7
	200	1 1/8	6 1/2
	300	1 1/16	6
	450	15/16	5 1/4
	600	7/8	5
	750	13/16	4 3/4
	900	13/16	4 1/2
	1200	3/4	4
	1500	11/16	3 3/4
8	50	1 3/4	7 1/2
	100	1 1/2	6 1/2
	150	1 3/8	5 3/4
	200	1 1/4	5 3/8
	300	1 1/8	5
	450	1 1/16	4 1/2
	600	15/16	4 1/4
	750	15/16	3 7/8
	900	7/8	3 3/4
	1200	13/16	3 1/2
	1500	3/4	3 1/4

Horsepower to be Transmitted	Double-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
10	50	2	6 3/4
	100	1 5/8	5 1/2
	150	1 1/2	5
	200	1 7/16	4 3/4
	300	1 1/4	4 1/4
	450	1 1/8	3 3/4
	600	1 1/16	3 1/2
	750	1	3 1/4
	900	15/16	3 1/8
	1200	7/8	2 7/8
	1500	13/16	2 3/4
12	50	2 1/8	6
	100	1 3/4	5
	150	1 5/8	4 1/2
	200	1 1/2	4 1/4
	300	1 3/8	3 3/4
	450	1 1/4	3 3/8
	600	1 1/8	3 1/8
	750	1 1/16	3
	900	1	2 3/4
	1200	15/16	2 5/8
	1500	7/8	2 1/2
	...	...	...

cooling properties of graphite are too well-known to require discussion. Anyone who has had trouble with worm gearing will find this lubricant well worth trying.

Horsepower of Worm Gearing. — A great many manufacturers of worm-gear drives in Europe build and guarantee drives for a given horsepower at a given speed. The accompanying tables give the horsepower that may be safely transmitted by worm gearing at given speeds of the worm. It is important in worm drives to keep the diameter of worm as small as possible. The tables, therefore, give the diameter of the largest allowable pitch diameter of the worm. It is not advisable to make the worm larger, as the gearing may then run hot and start to cut, but there is no objection to making the pitch diameter smaller, if the

Horsepower Transmitted by Worm Gearing, Triple-threaded Worm
(Gear: Phosphor-bronze; Worm: Hardened Steel)

Horse-power to be Trans-mitted	Triple-threaded Worm			Horse-power to be Trans-mitted	Triple-threaded Worm			Horse-power to be Trans-mitted	Triple-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches		Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches		Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
4	800	5/8	7¾	10	100	1½	7½	16	100	1¾	5¾
	1000	9/16	7½		150	1⅜	6¾		150	1⅝	5
5	400	13/16	8¼		200	1¼	6¼		200	1½	4½
	500	13/16	7¾		300	1⅛	5¾		300	1⅜	4¼
	600	¾	7½		400	1 1/16	5¼		400	1¼	4
	800	11/16	6¾		500	1	5		500	1 3/16	3¾
	1000	⅝	6¼		600	15/16	4¾		600	1⅛	3½
6	300	15/16	7¾		800	⅞	4¼		800	1 1/16	3¼
	400	⅞	7½		1000	13/16	4		1000	1	3⅛
	500	13/16	7⅛	12	50	1⅞	8	20	30	2⅝	6¾
	600	13/16	6½		100	1⅝	6¾		50	2¼	5¾
	800	¾	6		150	1 7/16	6		100	1⅞	5
	1000	11/16	5¾		200	1⅜	5⅝		150	1¾	4⅜
8	150	1¼	8		300	1¼	5		200	1⅝	4⅛
	200	1⅛	7¼		400	1⅛	4½		300	1½	3¾
	300	1 1/16	6½		500	1 1/16	4¼		400	1⅜	3½
	400	1	6		600	1	4		500	1 5/16	3¼
	500	15/16	5¾		800	15/16	3⅞		600	1¼	3
	600	⅞	5½		1000	⅞	3¾		800	1⅛	2⅞
	800	13/16	5⅛	16	30	2⅜	7½		1000	1 1/16	2¾
	1000	¾	4⅞		50	2⅛	6¾		...	...	...

linear pitch of the worm is not too coarse to prevent this. In many cases in the tables the pitch is so coarse with relation to the pitch diameter that it would not be possible to have a separate worm mounted on a shaft, but the worm must be made an integral part of the worm-shaft. Where there is a choice between using single, double or triple threads for the same drive, it is preferable to use double or triple threads. The tables apply to phosphor-bronze worm-wheels and hardened steel worms. If the worm-wheel is made of cast iron, instead of phosphor-bronze, the pitch should be made $1\frac{1}{3}$ greater than that given in the tables.

In the case of unfinished teeth, the pitch diameter of the worm should be only 0.8 times that given in the tables.

The best material for worm gearing is hard phosphor-bronze for the worm-wheel and hardened steel for the worm. The next best materials are cast iron for the worm-wheel and hardened steel or cast iron for the worm. Steel or steel castings for

Horsepower Transmitted by Worm Gearing, Quadruple-threaded Worm
(Gear: Phosphor-bronze; Worm: Hardened Steel)

Horse-power to be Trans-mitted	Quadruple-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
6	600	¾	8
	750	1/16	7½
8	300	15/16	8½
	400	15/16	7¾
	500	⅞	7¼
	600	13/16	7
	750	¾	6½
10	200	1 3/16	7¾
	300	1 1/16	7
	400	1	6½
	500	15/16	6
	600	⅞	5¾
	750	13/16	5½
12	100	1½	8½
	150	1⅜	7½
	200	1¼	7
	300	1⅛	6¼

Horse-power to be Trans-mitted	Quadruple-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
12	400	1 1/16	5¾
	500	1	5½
	600	15/16	5¼
	750	⅞	5
16	75	1¾	7½
	100	1⅝	7
	150	1½	6¼
	200	1 7/16	5¾
	300	1 5/16	5¼
	400	1 3/16	4⅞
	500	1⅛	4⅝
	600	1 1/16	4⅜
	750	1	4
20	50	2⅛	7
	75	1⅞	6½
	100	1¾	6
	150	1⅝	5½

Horse-power to be Trans-mitted	Quadruple-threaded Worm		
	Rev. per Minute of Worm	Linear Pitch of Worm, Inches	Max. Pitch Diam. of Worm, Inches
20	200	1½	5
	300	1⅜	4½
	400	1¼	4¼
	500	1 3/16	4
	600	1⅛	3¾
	750	1 1/16	3⅝
25	25	2¾	7½
	50	2⅜	6¼
	75	2⅛	5½
	100	2	5¼
	150	1¾	4¾
	200	1⅝	4⅜
	300	1½	4
	400	1 7/16	3¾
	500	1 5/16	3½
	600	1¼	3¼
	750	1 3/16	3⅛

both the worm-wheel and worm are only allowable for slow speeds. The teeth in the worm-wheel and the thread on the worm should always be cut, whenever the gearing is to be used steadily or at a reasonably high speed.

These tables are based upon the practice of a prominent European manufacturer making worm drives guaranteed to transmit given amounts of power. Information on this subject is extremely scarce and no tables have previously been published in this form giving the horsepower transmitted by worm gearing, except in Machinery's Handbook, from which these tables are reproduced.

CHAPTER VIII

THE DESIGN OF SELF-LOCKING WORM-GEARS

THE old opinion that the friction and wear of worm-gears are necessarily very great, and that the efficiency is necessarily very low, making worm gearing an unmechanical contrivance, is not as frequently met with now as formerly. Unwin states that in well-fitted worm gearing, of speed ratios not exceeding 60 or 80 to 1, motion will be transmitted backwards from the wheel to the worm. In Prof. Forrest R. Jones' work on machine design may be found tabulated the results of many examples from practice, some of which show an efficiency as high as 74 per cent before abrasion began, the most notable example being that of a worm running at a surface speed of 306 feet per minute under a load of 5558 pounds, and showing an efficiency of 67 per cent, with no abrasion. The tables in Professor Jones' work show that under light loads very high surface or rubbing speeds are allowable, running as high as 800 feet per minute. It has also been pointed out that an increase in the thread angle, in general, increases the efficiency.

There is, however, an important function of worm gearing which is not, as a rule, brought out adequately by writers on worm gearing, and which in certain classes of machinery is of the first importance, often, indeed, becoming the determining factor in deciding upon the choice of a worm-gear as the power transmitter. It is the property a worm-gear possesses, under certain conditions dependent upon its design, of being self-locking, and preventing motion backwards.

An instance where this property becomes of prime importance and accounts for the use of the worm-gear is in crane work, where the winding drum is driven by a worm-gear so designed that, when the power is shut off, the gear will not run down or backwards under the impulse of the load, but will be self-locking, holding the load at any point.

Fig. 1 shows a single-thread worm in mesh with the worm-wheel, a being the angle of the worm thread with the axis of the worm-wheel, and in order that the system may be self-locking, that is, that the worm-wheel may be unable to run the worm, the tangent of the angle a must be less than the coefficient of friction between the teeth of the worm and wheel, or as

$$\tan a = \frac{p}{\pi d}, \quad \text{so} \quad \frac{p}{\pi d} < f \qquad (1)$$

in which p = the pitch; d = the pitch diameter of the worm; and f = the coefficient of friction between the worm and wheel.

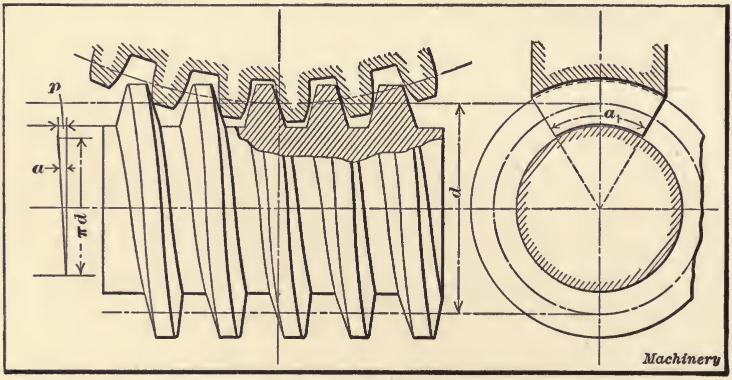


Fig. 1. Single-threaded Worm in Mesh with Worm-wheel, used to Illustrate the Principle of Self-locking Worm Gearing

It is necessary to assume a value for f, which, if the condition of determining the use of the worm-gear is its self-locking property, should be assumed conservatively low. Unwin states under the authority of Professor Briggs that a well-fitted worm-gear will exhibit a lower coefficient of friction than any other kind of running machinery. Professor Jones gives a series of values for the coefficient of friction of screw gears, one of which is a pinion of 4 inches pitch diameter, the average value being $f = 0.05$, corresponding to a rubbing velocity of 250 feet per minute. Mr. Halsey assumes $f = 0.05$, and Mr. Wilfred Lewis says that when the worm-gear is worked up to the limit of its safe strength, a rubbing velocity greater than 200 to 300 feet per minute will prove bad practice. It is in heavy machinery where worm-gears are mostly used as self-locking transmission elements, and here they are usually worked up to the safe strength of the wheel; hence it is fair to assume $f = 0.05$ when designing a self-locking worm-gear, and to limit the rubbing velocity to 200 feet per minute, and we have for the limiting value of p at which the system will be self-locking:

$$p = 0.05\,\pi d = 0.157\,d \tag{2}$$

The sliding velocity in feet per minute at the pitch line is expressed by

$$V = \frac{\pi\,dn}{12} = 0.262\,dn \tag{3}$$

where d = the pitch diameter of the worm, and n = the number of revolutions per minute of the worm.

Under the above assumption, that for continuous service and heavy pressures the sliding velocity should not be more than 200 feet per minute, we have as the limiting value of d to avoid all cutting:

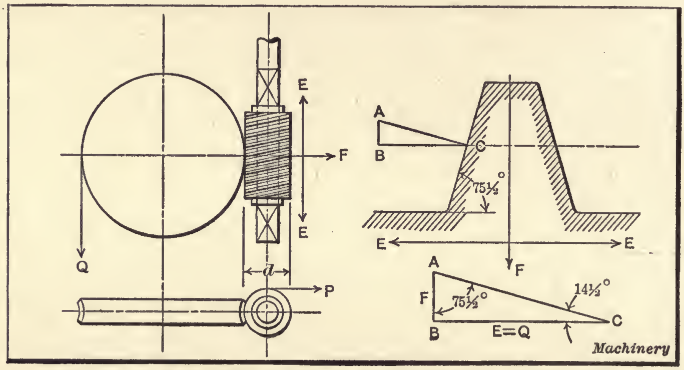
$$d = \frac{200}{0.262\,n}$$

The exact nature of the surface of contact between a worm and wheel is involved in doubt; many claim it is only a point; it certainly is not large, and consequently a wide face for the wheel is not needed.

If the angle a_1 is made 60 or 75 degrees, it will make the face satisfactory for any ordinary worm of from 4 to 6 inches in diameter.

There is in all worm gearing a very heavy end-thrust on the worm-shaft, and also an outward force normal to the worm-axis, each of which must be suitably provided for in the design of the shaft and bearings. The end-thrust may be taken by bronze washers slipped into the bearings at the end of the shaft, which may be removed when worn and replaced with new ones. Shoulders may be provided on the shaft, between which and the bearings bronze collars may be placed, these being split to enable new ones to be easily and quickly placed in position when the old ones become worn. Roller thrust bearings are very often applied to worms, and these as well as the bronze washers may be supplied with adjusting set-screws to take up the wear, instead of renewing the washers.

In Fig. 2 let P = the tangential force at the pitch line of the worm, d = the pitch diameter of the worm, Q = the tangential force at the pitch line of the worm-wheel, E = the end-thrust of



Figs. 2 and 3. Diagrams for the Derivation of Formulas

the worm-shaft, and F = the force on the worm-shaft normal to the worm-axis; then, friction being neglected:

$$Q = \frac{P\pi d}{p} \qquad (4)$$

In Fig. 3 draw line BC parallel to the axis, or coinciding with the pitch line, of the worm; let this line represent the force $E = Q$; draw AB normal to this line; it will then also be normal to the axis of the worm; then, when measured to the same scale to which BC is drawn, $AB = F$; if the angle CAB is $75\frac{1}{2}$ degrees, we have:

$$\frac{F}{Q} = \tan 14\tfrac{1}{2} \text{ degrees} \qquad (5)$$

$$F = 0.250\,Q \qquad (6)$$

Taking friction into consideration, the force P_1, tangential to the pitch line of the worm, which it is necessary to employ in order to produce a force Q tangential to the pitch line of the wheel, is given by Weisbach as

$$P_1 = Q\frac{h+f}{1-hf} \tag{7}$$

in which

$$h = \frac{p}{\pi d}.$$

The efficiency of the worm and wheel is then

$$\frac{P}{P_1} = e. \tag{8}$$

Example. — A single-thread worm of 1-inch pitch, running 80 revolutions per minute, is to transmit to a worm-wheel a tangential force $Q = 5000$ pounds, and is to be self-locking.

From (3)

$$d < \frac{200}{0.262 \times 80}$$

or d may be as large as 9.5 inches before abrasion need be feared.

From (2), $p < 0.157\,d$; assume $p = 0.125\,d$; then, as $p = 1$ inch, $d = 8$ inches, or the worm will require to be 8 inches pitch diameter in order that the angularity of the thread may be small enough to make the system self-locking. It will be seen that the required diameter will be increased as the value of f is decreased, and in case the required diameter of the worm proves too great for practice, and the pitch cannot be reduced on account of considerations of strength, some outside aid, such as a brake or friction disk applied to the worm-shaft, will have to be adopted.

From (7) as

$$h = \frac{p}{\pi d} = \frac{1}{3.14 \times 8} = 0.04$$

we have

$$P_1 = 5000\frac{0.04 + 0.05}{1 - (0.04 \times 0.05)} = 451 \text{ pounds.}$$

From (4)

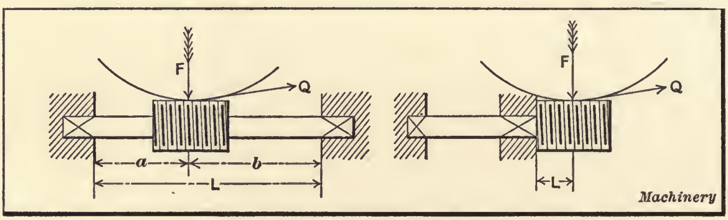
$$5000 = \frac{3.14 \times 8 \times P}{1}, \quad \text{or} \quad P = 199 \text{ pounds.}$$

From (8)

$$\frac{P}{P_1} = \frac{199}{451} = 44 \text{ per cent for the efficiency of the worm-gear.}$$

The formulas may, by starting with those for the efficiency, be used to determine the pitch diameter which will give the proper thread angle for any given pitch and degree of efficiency.

It is clear from the foregoing that a worm-gear of large pitch will require a pitch diameter of the worm altogether too large for practice, if it is to be self-locking, and that the system as usually designed may be expected to run backwards. To prevent this a friction disk may be placed in the bearing which receives the thrust of the worm-shaft when the system is running backwards, and the diameter of the disk so proportioned as to just hold the worm-shaft stationary under the impulse of the worm-wheel.



Figs. 4 and 5. Diagrams for Determining Bearing Friction

Bearing Friction. — The foregoing discussion neglects the effect of the thrust of the worm-shaft in its bearings, the frictional resistance of which must be added to that of the teeth to obtain the actual conditions of a self-locking system. This frictional resistance depends upon the values of the end-thrust E and the normal force F already found, and the diameter and form of the bearing. In nearly all cases of worm gearing the mounting of the worm upon the shaft will be covered by one of three cases, either unsymmetrically between the bearings, symmetrically between the bearings, or overhung.

In Case 1, Fig. 4, the bending moment upon the worm-shaft is

$$M = \frac{Fab}{L} = \frac{0.250\,Qab}{L} \qquad (9)$$

In Case 2, same as Case 1, except that the worm is central between the bearings, and

$$a = b = \frac{L}{2}$$

the bending moment upon the worm-shaft is

$$M = \frac{0.250\,QL}{4} = 0.0625\,QL. \qquad (10)$$

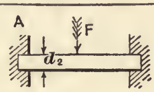
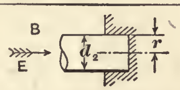
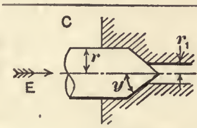
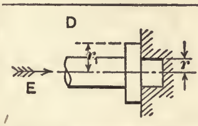
In Case 3, Fig. 5, the bending moment upon the worm-shaft is

$$M = FL = 0.250\,QL. \qquad (11)$$

In each of the above cases the shaft is subjected to a combined twisting and bending strain, the twisting moment being the same in each case, $T = PR$, which is, however, so small as to be negligible in what follows.

In the following table the first column shows the several styles of journals most commonly used for worm-shafts, the second

Table giving Moment of Friction with Various Types of Bearings

Style of Journal	Moment of Friction	f	$\frac{\text{Moment of Friction}}{R}$
	$\frac{fFd_2}{2}$	0.05	$\frac{0.04\,Pd_2}{p}$
	$\frac{2fEr}{3}$	0.05	$\frac{0.2\,Pr}{p} = \frac{0.1\,Pd_2}{p}$
	$\frac{2fE(r^3 - r_1^3)}{3r \sin y}$	0.05	$\frac{0.2\,P(r^3 - r_1^3)}{pr \sin y}$
	$\frac{2fE(r_1^3 - r^3)}{3(r_1^2 - r^2)}$	0.05	$\frac{0.2\,P(r_1^3 - r^3)}{r_1^2 - r^2}$

column gives the moment of friction for each under a load in the direction of the arrow, the third column gives the coefficient of friction assumed, and the fourth column gives the tangential force P_2 at the pitch line of the worm, resulting from the resistance of friction in the journals, and found by dividing the moment of friction in Column 2 by the pitch radius of the worm.

There are always acting upon the worm-shaft the two forces F and E; consequently to get the resultant retarding force tangential to the pitch line of the worm, we must take the sum of the resultants due to the frictional resistance of each force separately. Referring to the table, for each worm-shaft, find the conditions shown at A, in addition to the conditions shown either at B, C or D, as the case may be, and the total resultant force P_2 at the worm pitch line will be the sum of the quantities given in Column 4 opposite the particular cases.

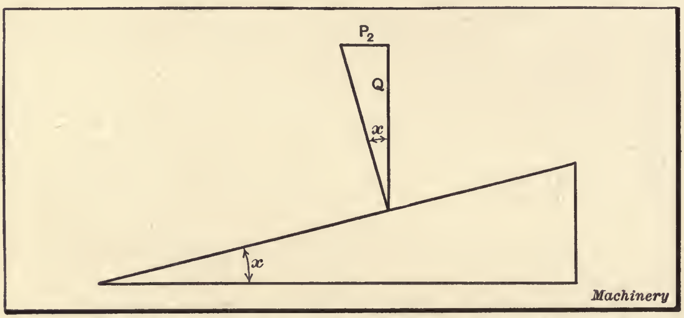


Fig. 6. Diagram for Determining the Angle of Repose Corresponding to the Journal Friction

These frictional resistances developed by the journals act in a direction helpful to the self-locking property of the worm, and enable the designer to use a larger thread angle for a given diameter of worm, or a smaller diameter of worm for a given thread angle, thus keeping within the limits of good practice, and increasing the efficiency of the system for the forward movement.

Having determined the force P_2 tangential to the worm pitch line, resulting from the frictional moment at the journals, the angle of repose for this force acting with the force Q, as shown in Fig. 6, is given by the equation,

$$\tan x = \frac{P_2}{Q}$$

The thread angle found previous to the consideration of the effect of the journal friction may now be increased by the angle

x, making the thread angle $a + x$. This may be accomplished either by increasing the thread angle, increasing the pitch, or decreasing the pitch diameter.

Consider, now, that in the foregoing example, the worm-shaft is of the form in Case 2, the worm being central between the bearings, and the distance between bearings being 36 inches.

Then, from (5), we have:

$$F = 0.250 \times 5000 = 1250 \text{ pounds.}$$

From (10)

$$M = \frac{0.250 \times 5000 \times 36}{4} = 11,250 \text{ inch-pounds.}$$

Assuming $s = 10,000$ pounds per square inch for the allowable fiber stress in the worm-shaft, we have:

$$M = \frac{\pi}{32} d_2^3 s, \quad \text{or} \quad d_2 = 2.28 \text{ inches.}$$

From the table, Case A,

$$P_1 = \frac{0.04 \times 199 \times 2.28}{1} = 18.15 \text{ pounds.}$$

From the table, Case B,

$$P_1 = \frac{0.1 \times 199 \times 2.28}{1} = 45.37 \text{ pounds.}$$

Then

$$P_2 = 18.15 + 45.37 = 63.52 \text{ pounds.}$$

$$\tan x = \frac{63.52}{5000} = 0.0127$$

$$x = 0 \text{ deg. } 44 \text{ min.}$$

From (1)

$$\tan a = \frac{1}{3.14 \times 8} = 0.04$$

$$a = 2 \text{ deg. } 17 \text{ min.}$$

Then

$$a + x = 3 \text{ deg. } 1 \text{ min.}$$

$$\tan 3 \text{ deg. } 1 \text{ min} = 0.053$$

$$\frac{p}{\pi d} = 0.053 = h, \text{ and } d = 6 \text{ inches, approx.}$$

If, now, we substitute these new values of h and d in Equations (7) and (4), we continue as follows:

From (7)

$$P_1 = 5000 \frac{0.053 + 0.05}{1 - (0.053 \times 0.05)} = 516 \text{ pounds.}$$

From (4)

$$5000 = \frac{P \times 3.14 \times 6}{1}, \text{ or } P = 265 \text{ pounds.}$$

From (8)

$$\frac{P}{P_1} = \frac{265}{516} = 51 \text{ per cent efficiency for the worm-gear.}$$

The total efficiency of the system, taking account of the journal friction, will be:

$$\frac{P}{P_1 + P_2} = \frac{265}{516 + 63.52} = 46 \text{ per cent.}$$

It thus becomes clear that while the efficiency of the worm threads and wheel teeth has been increased above 50 per cent, the efficiency of the whole system, including the journals, is below 50 per cent, and the system retains its self-locking property. It is evident that when running forward, the end-thrust E upon the worm-shaft will be upon the opposite end from that when running backward, and on this account a system may be designed to have a high efficiency on the forward movement and still preserve its self-locking property.

If both the journals have roller bearings, and the end taking the thrust on the forward movement has a ball bearing, while the opposite end be made like Case C or D in the table, properly proportioned, the worm may be designed to show a very high efficiency on the forward movement, while the frictional resistance of the step bearing on the opposite end will cause the system to be self-locking by reason of the energy absorbed at the step bearing.

General Method of Procedure. — The formulas may be put into more convenient form for this purpose, as follows: The designer will have, to start with, a knowledge of the force Q required at the worm-wheel, the force P_1 at the pitch line of the worm, developed from the source of power, the pitch required for the worm-wheel, and the efficiency e for which he wishes to design the system. We then have:

$$\frac{P}{P_1} = e, \text{ and } P = P_1 e.$$

Substituting this value for P in Equation (4) and solving for d, we have:

$$d = \frac{pQ}{3.14 P_1 e}$$

for the worm, neglecting the journals, when the journals and thrust bearings are roller and ball bearings, respectively, and

$$d = \frac{pQ}{3.14 (P_1 - P_2) e}$$

when the journals and thrust bearings are considered.

The worm being thus designed for the given efficiency on the forward movement, it remains to determine such proportions of the step bearing for the backward movement as will present enough frictional resistance to render the system self-locking. Let e_1 = the efficiency when the journals and thrust are considered, then:

$$\frac{P}{P_1 + P_2} = e_1, \quad \text{or} \quad P = e_1 (P_1 + P_2)$$

and substituting the value of P found above,

$$eP_1 = e_1 P_1 + e_1 P_2$$

and

$$P_2 = \frac{P_1 (e - e_1)}{e_1}$$

By equating this force P_2 to the proper quantity from Column 4 in the table of journal resistances, the proportions required of the journal or step bearing may be determined.

CHAPTER IX

THE HINDLEY WORM AND GEAR

The Hindley type of worm-gear was first used in Hindley's dividing engine, and was, by the inventor, considered superior to the ordinary type, in wearing quality. Investigation has practically settled that the nature of contact between the worm thread and the teeth of the ordinary worm-wheel is that of line contact, extending across the tooth on the pitch line. It has also been fairly well proved in practical examples that the contact is of a broader nature on account of the elasticity of the materials used in the construction. The convex surfaces of contact are flattened considerably under pressure and thus for practical purposes make actual surface contact. The contact in the ordinary worm and worm-wheel type is limited to two teeth of the wheel and worm thread, at most.

Comparison of Ordinary and Hindley Worm Gearing. — The conditions are much different in the case of the Hindley worm, and it is the intention in this chapter to show wherein the difference lies. As this style of gearing is not very common, a few words regarding its construction will not be out of place. Fig. 1 illustrates the Hindley worm, showing the theoretical form. This worm is not of cylindrical shape, but is formed somewhat like an hour-glass, after which it is sometimes named. The worm blank, being made smaller in diameter in the middle than at either end, conforms to the circumference of the wheel with which it meshes. The worm thread is cut by a tool which moves in a circular path about a center identical with the axis of the wheel with which it is to mesh, and in the plane in which the axis of the worm lies. The process is similar to ordinary thread cutting in the engine lathe, except for the difference in the path of the tool, the tool having a circular instead of a straight path.

It is evident that the worm shape is dependent on the particular wheel with which it is to run, and Hindley worms are not interchangeable with any other but an exact duplicate. That is, a worm cut for a Hindley gear of 50 teeth cannot be used successfully with a wheel of 70 teeth, although the pitch of the teeth is exactly the same. In the ordinary type of worm gearing one worm may be made to run with any number of diameters of wheels of the same pitch, and hobbed with the same hob.

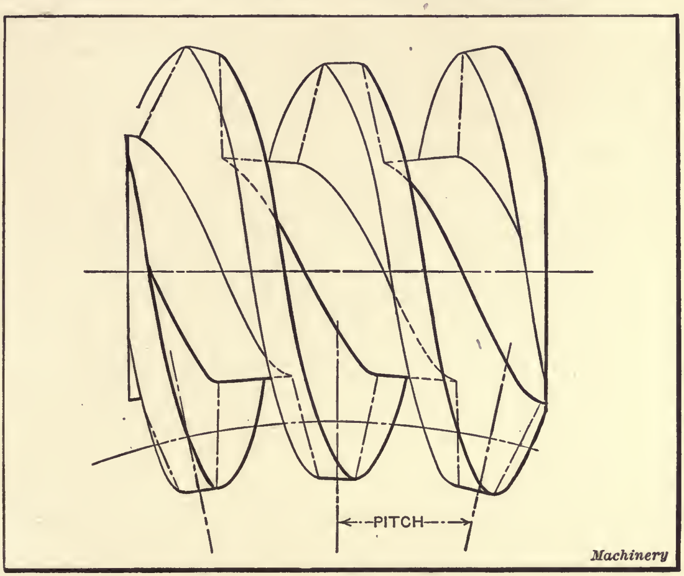


Fig. 1. Typical Hindley Worm

In action the two styles of worm-gear differ greatly, and both diverge widely in action from the case of a plain nut and screw, which may be taken to represent a worm and worm-gear, the latter of infinite diameter and with an angle of embrace of 360 degrees. In studying the action between the thread and teeth of the ordinary type of worm-gear, we must understand odontics, rolling contacts and the theory of tooth gearing in general, in order to understand the action of the ordinary worm-gear. But, in studying the action of the Hindley type, we are concerned with no such theories, as the action is purely sliding and devoid of rolling contact. In the ordinary worm we have an axial pitch which is constant from top to root of the thread, while in the Hindley worm we have a section in which the pitch of the thread varies from top to bottom.

The interference in the ordinary type of worm-gear is absent from the Hindley type, and the consequent undercutting and weakening of the teeth, therefore, is a feature with which the designer of the Hindley worm gearing does not have to contend. For this reason we are not limited in the length of teeth, by interference, as in the ordinary case. This fact permits a wide latitude in the choice of tooth shapes and proportions. In most examples we will find that the depth of thread is much greater in proportion to the thickness than in the ordinary worm-gear,

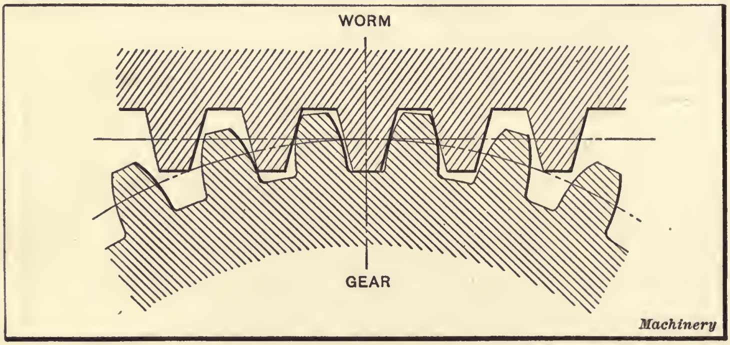


Fig. 2. Section of Ordinary Worm and Worm-wheel on Central Plane

in which the height is limited by reason of the interference at the top and root of the teeth.

Nature of Contact of Hindley Worm Gearing. — The general idea of the Hindley worm gearing is that there is surface contact between the worm and gear, and that the contact is generally over the whole number of teeth in mesh. If such were the actual conditions, the Hindley type would surely be an ideal mechanism for high velocity ratios, but that such is not the fact is the purpose of this treatise to point out. That the contact is of a superior nature we will not deny, nor that it is much nearer a surface contact than exists in the ordinary worm-gear. As a means of comparison Figs. 2 and 3 are shown. Fig. 2 shows an axial section taken through the worm and gear of the ordinary type, while Fig. 3 shows a similar section through the Hindley worm and gear. The "airy" appearance of Fig. 2 as compared with Fig. 3 indicates a vast difference in the nature of contact, and gives the advantage to the Hindley type, wherein is the origin of certain false ideas in favor of the latter. These illustrations also show peculiar differences in the action of the two types. The absence of rolling action in Fig. 3 is the most prominent, and it shows the similarity between this type of gear and a screw and nut.

From an inspection of Fig. 3 we may feel sure that the contact on the axial plane is as shown, but as to the nature of contact in a plane either side of the middle plane we are in the dark so

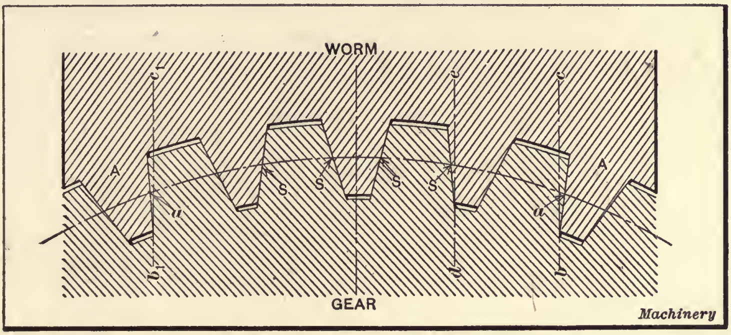


Fig. 3. Section of Hindley Worm and Gear on Central Plane

far as the drawing illustrates. Mr. George P. Grant says concerning the contact of the Hindley worm and gear: "It is commonly but erroneously stated that the worm (Hindley) fits and fills its gear on the axial section. . . . It has even been stated that the contact is between surfaces, the worm filling the whole gear tooth. . . . It is also certain that it (the contact) is on the normal and not on the axial section, and that the Hindley hob will not cut a tooth that will fill any section of it. The contact may be linear on some line of no great length, but it is probably a point contact on the normal section."

It is not clear what reason Mr. Grant had for saying that the contact is normal instead of axial, because there seems to be good reason to believe that the contact is on the axial section since it is on this section that the teeth of the hob have a common pitch. The teeth have not a common pitch on any section at an angle with the axial section. For what reason would one expect to find contact on the normal section in this case any more than in the case of the ordinary worm? Since both styles of worm-wheels are hobbed with a revolving hob which lies in a plane perpendicular to the axis of the worm-wheel, the contact could hardly be on a normal section.

Professor MacCord states that he considers the contact to be line contact on the axial section, and he gives directions for obtaining the exact nature of the contact and also the thread and tooth sections. These directions, on account of the complicated nature of the method, are difficult to follow. Much, however, can be found out by simple methods. In what follows, describing these simpler methods, the results, of course, are of an

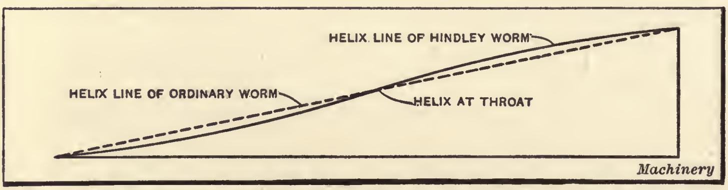


Fig. 4. Development of Ordinary Worm and Hindley Worm Spirals on a Plane

approximate order, but they nevertheless give a means of comparison and a material basis for the line of argument.

The Ideal Case Considered. — It is assumed that we are examining an ideal Hindley gear in which the worm and wheel are theoretically correct in shape and that the surfaces are perfectly smooth and inelastic. From the nature of the worm, the helix angle varies from mid-section to the ends, decreasing as the thread approaches the ends of the worm. The thread is spiral as well as helical. This change in the thread angle is caused by the increase in diameter at the ends of the worm and by the fact that the axial pitch of the thread decreases as it reaches the ends. The decrease in axial pitch is due, of course, to the circular path of the threading tool. If we take a development on a flat surface of a line scribed in the spiral path on the worm blank, as shown in Fig. 4, the change in the angle becomes noticeable.

In the operation of forming the teeth of the gear, the blank is rotated, each portion of the hob working the tooth into shape so that it will pass the corresponding portion of the worm thread without interference, permitting a smooth transmission of motion. If each portion has a different shape or is placed in a different relation, the shape of a gear tooth will be a compromise between the extremes, and this is what is actually the result, as we shall see later.

The progressive steps of the process are shown in Fig. 5; the successive positions of one tooth are shown, beginning at the left and ending at the right-hand position where each tooth is

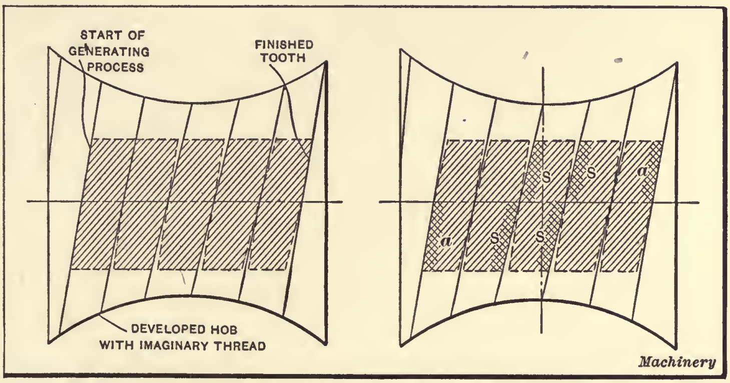


Fig. 5. Successive Steps in Shaping the Hindley Worm **Fig. 6. Surfaces of Contact of the Hindley Worm**

given its final shape. The nature of the process is shown in Fig. 6, the shaded portions representing the gear teeth. Here we have a representation of the contact of the thread and teeth; it shows that surface contact is impossible on any but the heavily shaded portions of the teeth, it being confined to the mid-section and the extreme end sections of the worm. Line contact is obtained throughout the length of the worm on the axial plane. This figure also shows that no advantage is gained in surface contact by making the worm of greater length. The location of the contacts are shown in Fig. 6, at *a*, *s*, *s*, *s*, *s*, *a*, but it must be remembered that they lie on opposite sides of the cutting plane. From this it is apparent that the worm does not entirely fill the space between the teeth of the gear and that the contact is not wholly a surface contact.

Let us investigate still further and see whether the conditions are not modified by other irregularities: Fig. 7 is drawn to represent a worm and gear of the Hindley type, in mesh, the teeth of which have no depth. As before mentioned, the peculiarity of this type of worm is its hour-glass shape. The hob and worm may be treated as identical in form. In the process of generation, the tooth has a pitch line curvature that changes with corresponding positions in relation to the thread portion acting upon

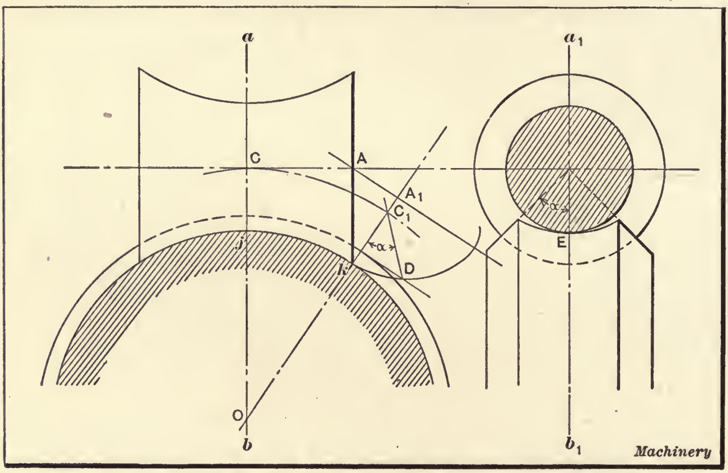


Fig. 7. Effect of Hour-glass Shape on Worm-wheel Contact

it. The tooth must necessarily be modified from what it should be for any particular location in its contact with the worm thread. It is quite clearly shown that if the tooth is to fill the worm thread or *vice versa*, it must be formed in strict accordance with the thread at that particular point. Thus if at *j* the tooth fills the thread, that tooth must be formed by the thread at that point, while the tooth at *k* must be formed by the thread at *k*. Now, since each tooth must pass from *k* to *j*, its form must be such that it will do so without interference. It is evident that the radial section of the gear at *k* must be the same as at *j*. Since the worm is largest in diameter at *k*, the curvature of the tooth on the radial section is dependent on the thread at that point. The curvature of the tooth at *k* evidently is that of an ellipse whose major axis is AA_1. Now, since the thread is made with angular sides, the hob could hardly act on the teeth of the gear the same at all points from *k* to *j* except on the axial plane where the relative shape of the hob thread is the same for any position along the line of action (see Fig. 3). This is evident from Fig. 7 at *E*, which point only touches at the mid-section of the worm. Therefore we still have the line contact from top to bottom of teeth on the axial plane, but the construction, Fig. 7, shows that the surface contact *s*, *s*, *s*, *s*, Figs. 3 and 6, does not actually exist, but that the surface contact at the ends of the worm remains undisturbed.

From the above we may safely conclude that the hob at *j* has but little effect on the actual shape of the tooth, and that its influence increases until *k* is reached. Fig. 7 also shows a good reason why the contact may be considered axial instead of normal, by the mere fact of the differences in curvature of worm and wheel at any point other than *k*. In practice the contact may appear to be surface contact, but this, no doubt, is due to the influence of the lubricating oil and the fact that materials of construction are distorted to some extent in form when subjected to pressure. This distortion permits the worm thread to imbed itself into the worm-wheel teeth, somewhat broadening the contact for the time being. The conditions as stated in the above discussion would be met in the case of a hardened worm and gear with surfaces finished by lapping. In practice the worm and gear are ground together, sand and water being used as the abrasive. This grinding wears down the roughness of the surfaces and tends to correct irregularities in form that develop in the hobbing process.

Objections to the Hindley Gear. — The objections to the Hindley type of worm-gear are many and are widely known. It must be set up accurately, the alignment being made perfect. End play is a feature that must be avoided, as any longitudinal displacement of the worm will cause the gear to cut. These peculiarities are the greatest drawbacks to the use of this gear, and because of them the author believes that it will not come into common use, at least not so common as the worm drive of the ordinary type. This opinion is strengthened by the fact that we have become so much more familiar with the latter type as to be able to design and construct drives that work satisfactorily in every respect.

Modifications of Hindley Worm-gear Practice. — Some modifications have been made in the process of manufacturing the Hindley worm-gear. One that is probably of first importance is that known as the "second cut." The effect of the second cut is indicated in Fig. 8. From this illustration one would say that the object of the second cut is to remove the points of contact. Whether this is the reason or not, it is a fact that it does remove

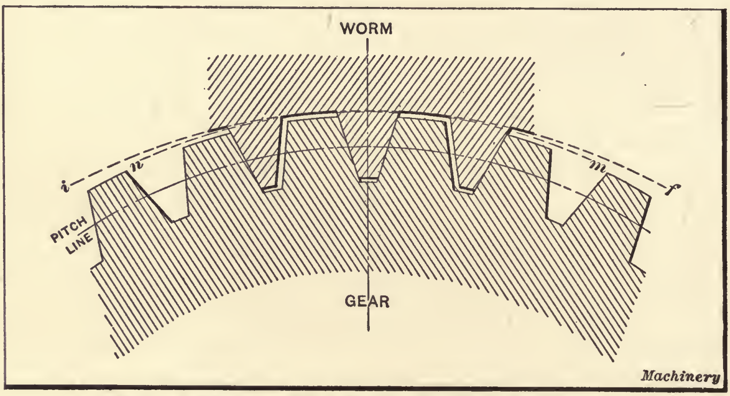


Fig. 8. Effect of the "Second Cut" on Contact

considerable of the contact from all but the mid-section of the worm. This second cut is made by enlarging the diameter of the circle in which the threading tool travels when cutting the worm. It is said to have advantages that add to the wearing quality of the drive, but just what these advantages are is not apparent, and since the process is considered more or less a trade secret, it is difficult to obtain authentic reasons for its use.

The limiting length of the worm is dependent on the shape of the thread. In Fig. 8 the worm is shown with three teeth in mesh, while Fig. 3 shows five. Fig. 3 shows a case that would be impossible in practice on account of the undercut teeth *A* which lock the worm in mesh. The side of the thread must fall inside the line *bc* to permit the worm and gear to be assembled.

Conclusions Regarding the Hindley Worm and Gear. — The following are the conclusions, derived from the investigation regarding the Hindley type of gear:

1. The contact is purely sliding contact.

2. The nature of the contact is linear, closely resembling surface contact.

3. Linear contact extends from the top to the root of the tooth.

4. The contact is on the axial section.

5. The thread section fills the tooth space on the axial section only.

6. The mid-portion of the hob has little or no effect in shaping the teeth of the gear.

7. Surface contact exists on opposite sides of the axial plane at the end of the worm thread and is intermittent in nature, because the end of the thread passes out of contact with the tooth in the revolving of the worm. This contact is on a plane normal with the thread angle.

In practice it is usual to allow considerable back-lash between the thread and the tooth of the worm and gear. This play tends to counteract bad workmanship, either in construction or erection.

CHAPTER X

METHODS FOR FORMING THE TEETH OF WORM-WHEELS

To correctly classify and comprehend the various methods and machines for cutting the teeth of worm-wheels, it is first necessary to clearly define the term "worm gearing." We will consider that by worm gearing we mean gearing of the type of which a cross-section is shown at the left of Fig. 1, in which the acting face of the wheel is curved to fit the form of the worm, and in which the whole width of the wheel face is in active working contact with the worm.

The action is best understood by taking vertical sections on the center line *A–A*, and other lines such as that at *B–B*, parallel with the center line. Sections on lines *A–A* and *B–B* are shown at the right of the cut. With worm gearing of standard form, the section on line *A–A* shows the worm to have the profile of an involute rack, while the teeth of the wheel show outlines identical with those of the corresponding involute gear of the same pitch and number of teeth, suited to engage with the rack. In other words, the teeth of the gear are such as would be formed by the teeth of the worm if the latter acted as a rack in a molding-generating operation. A section on line *B–B* shows that the teeth of the worm have a distorted outline on planes removed from the axial plane. If we consider these distorted teeth as the teeth of a rack, molding their mating tooth spaces in a gear running on the same center as the worm gear and at the same speed, it will form the distorted wheel teeth shown for the section on line *B–B*. In a word, each section of the worm parallel to the axial section *A–A* is a rack section, which molds in the wheel below it the proper teeth to mesh with it in accurate conjugate action. The true worm-wheel, it is thus seen, must be formed by the molding-generating process.

The same worm as that shown in Fig. 1 may be made to engage with a spiral gear of the same number of teeth as the worm-wheel, provided the teeth are of the proper pitch and set at an angle to agree with the helix angle of the worm. The action of such gearing, however, does not, like that in Fig. 1, take place on all sections *A–A*, *B–B*, etc., but is confined to a point at or near the center line *A–A*. The contact, in other words, is point contact, and not line contact extending clear across the face of the wheel. Such a combination, in fact, is not a case of worm gearing, but a case of spiral gearing — and a very poor case at that.

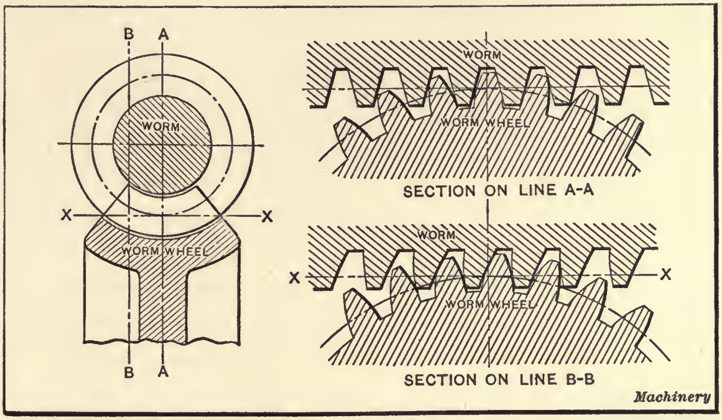


Fig. 1. Action of a True Worm-wheel

Gashing Worm-wheels by the Formed Cutter Process. — While the method of forming a true worm-wheel is thus seen to be accurately performed only by the molding-generating process, the accurate teeth produced by that process may be closely approximated in many cases by the "gashing" method, which belongs in the formed cutter classification. In this operation a milling cutter is used having approximately the outline of a normal section of the teeth of the worm to be used. This cutter is of the same diameter as the worm, and is set with relation to the axis of the work at the helix angle of the worm, as measured on the pitch line. It is centered over the wheel, and fed into the latter to the proper depth to form a tooth space; it is then drawn out again, the work is indexed to the next tooth space, and the cutter again sunk in to depth, the operation being repeated until the wheel is completed.

A universal milling machine is generally used for this operation. With the table set at 90 degrees, the cutter is first brought centrally over the work arbor by adjusting the saddle on the knee of the milling machine, and then the work is brought centrally with the cutter arbor by adjusting the table by the feed-screw. The work table is next swung to the helix angle of the worm which is to be used with the wheel. Then the cutting is proceeded with.

This gashing process gives a tooth very closely approximating the true tooth form, when the diameter of the worm is large as compared with the pitch, and when the worm is single-threaded, but, for multiple-threaded worms of smaller diameter in proportion to their pitch, the process is impracticable. This method is, however, used by at least one of the best-known builders of gear-cutting machines in forming the teeth in the index worm-wheel. It is used under the conditions which give a very close approximation to the true form of tooth, and is employed in this particular case for the sake of the high degree of accuracy obtainable. The index wheel is divided, in cutting, by a carefully-made and carefully-preserved master wheel. The step-by-step gashing process allows the *spacings* of this superior master wheel to be accurately reproduced in the index wheel being cut — more accurately, it is claimed, than would be possible if it were to be reproduced by the hobbing process.

The gashing process is also used for roughing out worm-wheels preparatory to hobbing. In a previously gashed wheel, as will be explained later, the hobbing operation is one of extreme simplicity, not requiring special machines or mechanisms of any kind.

The Molding-generating Principle. — As already explained, the molding-generating principle is the only one that will accurately *form* the teeth of worm-wheels. The principle involved is shown in Fig. 2. The forming worm (or hob) is connected by gearing with the worm-wheel blank to be formed, in the same ratio as in the finished worm gearing. While the blank and the forming worm are rotated together in this ratio, the latter is fed into the blank slowly, its threads forming the properly shaped tooth in the wheel. As the worm revolves, an axial section would give the appearance of a rack like that shown in section *A–A*, of Fig. 1, moving continuously and forming suitable gear teeth in the wheel below it. Any other section, such as *B–B* in Fig. 1, would also act as a distorted rack, forming correspondingly distorted gear teeth in that portion of the worm-wheel in the same plane.

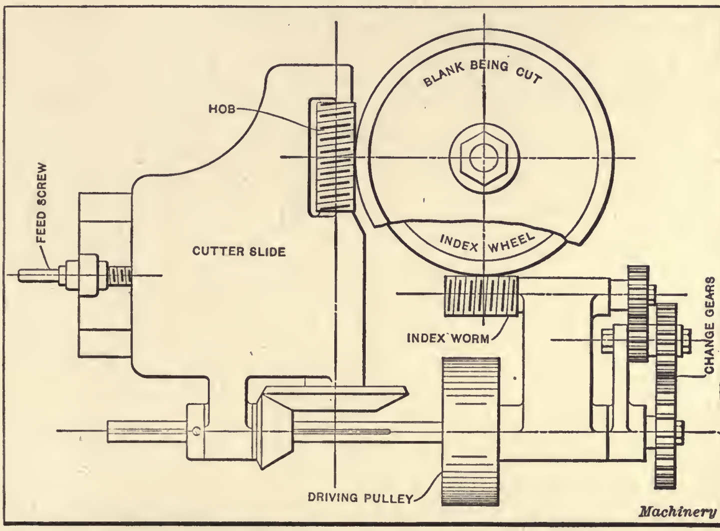


Fig. 2. Diagram showing the Principle of the Hobbing Process for Cutting Teeth in Worm-wheels

Of the various methods of operation, by which the molding-generating principle can be applied, shaping or planing is, of course, impracticable. Milling is the method generally employed. Grinding or abrasion is used to a limited extent, it being sometimes employed in the case of "grinding in" a worm with a wheel already roughly cut to shape. In this operation the worm and wheel are run together in place, under considerable pressure, the teeth of the gear being liberally supplied with oil and emery, which act as an abrasive and form the teeth of the gear and worm to fit each other.

In the commonly employed milling operation, the process is that known as "hobbing," and the milling cutter or tool used is a "hob." The hob, barring modifications required for relief or clearance, and allowance for regrinding, as explained in a following chapter, is practically a replica of the worm which is to be used, but with grooves cut in it so as to form teeth. This hob is rotated in the proper ratio with the work, exactly as shown in Fig. 2, and fed slowly down into it, cutting out the tooth spaces in the wheel as it does so. When it has reached the proper depth, the teeth are all formed to the proper shape.

Hobbing Worm-wheels in the Milling Machine. — The simplest method of rotating the hob and the work in the proper ratio with each other is that in which the work is first gashed, and then finished with the hob in such a way as to be driven by the latter, the work and the hob thus furnishing their own driving mechanism. The worm-wheel is mounted so as to revolve freely on dead centers. This is the simplest method of making correct worm-wheel teeth. It does not require special appliances of any kind, being done in an ordinary milling machine with a gashing cutter and a hob. Complete details of the practical operations for producing worm-wheels by gashing and hobbing will be given in the following chapter.

In cases where it is desired to hob worm-wheels directly from the solid without preliminary gashing, it is necessary to provide some special device for rotating the hob and the work in unison as in Fig. 2. Special worm-wheel hobbing machines are made for this purpose. One of the questions met with in this connection is the figuring of the gearing to properly connect the hob and the gear. This will be explained in the following paragraphs.

Gearing for Worm-wheel Hobbing Machines. — The manner in which the machine is geared will depend on the assortment of change gears with which it is provided. If there is a sufficient variety of these, simple gearing may be employed, using an idler on the swinging arm to connect the gear on shaft D with gear A, Fig. 3.

First find the revolutions of the hob for each revolution of the worm-wheel. This is found by dividing the number of teeth in the wheel by the number of threads in the hob, or worm, with which it is to run. For instance, if there are fifty teeth in the wheel and the worm is single threaded, the number of revolutions of the work to one of the hob will be 50 ÷ 1 = 50.

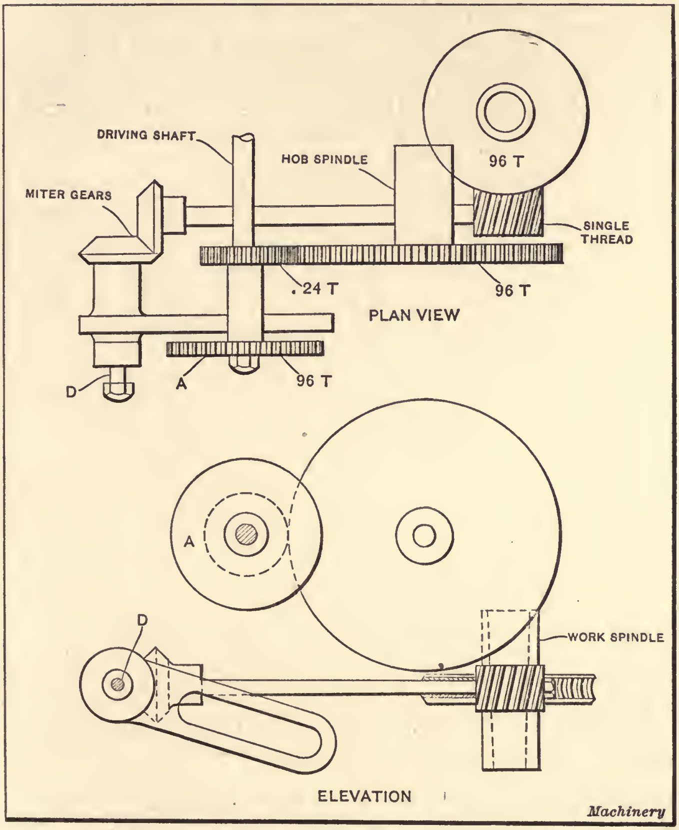


Fig. 3. Arrangement of Gearing in Hobbing Machine

This may be called the *ratio* of the wheel. If the worm is double threaded, the ratio will be 50 ÷ 2 = 25, and so on. With the gear connections indicated in Fig. 3, for simple gearing, when *A* has 96 teeth, the gear on shaft *D* must have a number of teeth equal to 4 times the ratio; that is to say, if we have a worm-wheel with 50 teeth, driven by a double-threaded worm, the ratio is

25, and the number of teeth in the gear at $D = 4 \times 25 = 100$. If a 48-tooth gear is used at A in place of the 96-tooth gear, the gear on D is found by multiplying the ratio by 2. If, for instance, the number of teeth in the wheel to be cut is 135, to mesh with a triple-threaded worm, the ratio will be $135 \div 3 = 45$ and the number of teeth for gear D will be $2 \times 45 = 90$, when A has 48 teeth.

A wider range of ratios can be provided for with a given number of change gears if A and D are connected by compound gear-

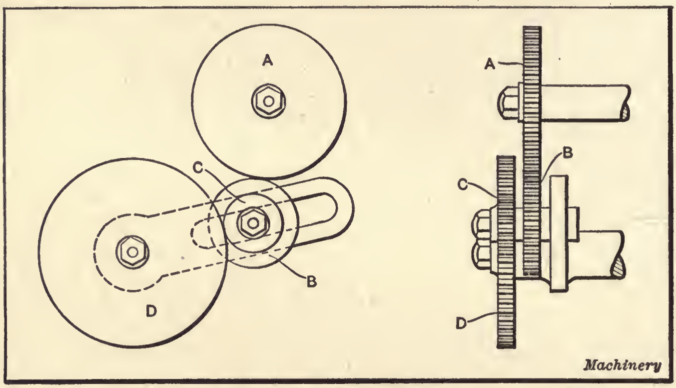


Fig. 4. Compound Gearing for Machine shown in Fig. 3

ing, as shown in Fig. 4, where A and C are the driving gears and B and D the driven gears. The rule for finding the number of teeth for B, C and D when A has 96 teeth then becomes:

$$\frac{\text{number of teeth in } B \times \text{number of teeth in } D}{\text{number of teeth in } C} = 4 \times \text{ratio.}$$

When A equals 48, this becomes:

$$\frac{\text{number of teeth in } B \times \text{number of teeth in } D}{\text{number of teeth in } C} = 2 \times \text{ratio.}$$

Suppose, for instance, that we have a set of change gears varying by 6, that is to say, the numbers run 18, 24, 30, 36, 42, etc., from 18 to 120. Suppose the ratio of the worm-gear to be cut is 50, and the number of teeth in gear A is 96; then we have:

$$\frac{\text{number of teeth in gear } B \times \text{number of teeth in gear } D}{\text{number of teeth in gear } C} = 4 \times 50 = 200.$$

By selecting a 60-tooth gear for *B*, a 120-tooth gear for *D* and a 36-tooth gear for *C*, we have:

$$\frac{60 \times 120}{36} = 4 \times 50.$$

Proving the calculations of the gearing for this machine is practically the same, whether simple or compound gearing is used. Consider that the whole mechanism is driven from the hob spindle; then the product of all the driven gears, divided by the product of all the driving gears, equals the number of teeth in the worm-wheel divided by the number of threads in the worm or hob. An idler gear between a driving and driven gear is not considered at all, as it has no effect on the motion other than to reverse it. Proving the first example by this method, we have:

$$\frac{96}{1} \times \frac{1}{1} \times \frac{100}{96} \times \frac{24}{96} = \frac{50}{2} = 25.$$

The number of teeth in these driving and driven gears are given in their order from the work, through the mechanism, to the hob. The fraction $\frac{1}{1}$ represents the miter gears, which are of even ratio, but the number of teeth of which are not given. For the last example the proof is similar. Here we have:

$$\frac{96}{1} \times \frac{1}{1} \times \frac{120}{36} \times \frac{60}{96} \times \frac{24}{96} = \frac{50}{1} = 50.$$

The Fly-tool Method of Cutting Worm-wheels. — By providing suitable driving and feeding mechanism, it is possible to use a simple fly-cutter for forming the teeth of worm-wheels in place of the expensive hob used in the operations previously described. The movements required for this method will be understood from a study of Fig. 5. Here is shown in dotted lines a worm meshing with a worm-wheel, a portion only of the periphery of which is seen. Such a worm, properly located with reference to a plastic blank and rotating with it in the proper ratio, will form accurate teeth in the latter by the molding-generating process. Gashing this worm makes of it a cutter by means of which the same form may be given to a blank of solid metal. The teeth of such a gashed hob coincide with the outlines of the thread of the worm.

In Fig. 5, in full lines, is shown a cutter bar with a blade T_1 of the same outline as the thread of the worm and the tooth of the corresponding hob. In order to permit this single cutting tool to perform the function of the worm as it molds the plastic substance, or of the hob as it cuts its shape in the metal, it must be fed helically as the bar and work revolve, following the outlines of the imaginary worm from one end to the other as the cutting progresses. Beginning at the left, for instance, the

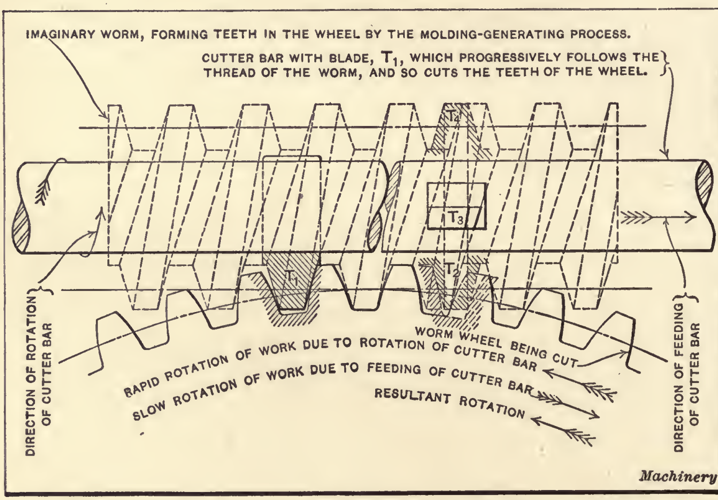


Fig. 5. Diagram showing the Principle of the Fly-tool Method of Cutting the Teeth of Worm-wheels

blade may be fed helically in the line of the thread, passing through positions T_1 and T_3, until the feed finally runs out at the extreme right.

The methods of giving this progressive helical change of position to the fly-cutter are various. It would be possible, for instance, to so connect the feed-screw, by which the cutter-bar is advanced with the rotating mechanism for the bar, through differential and change gearing, that a rotating movement due to the axial feeding of the latter would be added to or imposed upon the rotation due to its connection with the work, just as, in Fig. 11, Chapter IV, the rotation due to the downward feed of the cutter slide is combined with that due to the connection with the cutter spindle for rotating the work. If the proper change gears were selected so that, with the spindle- and work-driving mechanisms stationary, the feeding forward of the cutter bar would rotate the latter at the proper rate to give the lead of the work, the blade would evidently follow the path of the thread of the imaginary worm, as shown at T_1 and T_3 in Fig. 5. Owing to the action of the differential mechanism, it would still follow the thread of the imaginary worm, even if the latter, with the spindle- and work-driving mechanisms, were in motion.

Another method consists in combining in the work, also by differential gearing, a rotation due to the revolving of the cutter with a rotation due to the axial feed of the cutter-bar. That this produces the same effect as the previous arrangement will also be understood from Fig. 5.

First, let the rotation of the cutter be arrested. If the cutter-bar with a worm mounted on it, such as shown by the dotted lines, be now fed axially in the direction of the arrow, the positive connections between the feed and the work spindle, through the change gearing and the differential gearing, will cause the work to rotate uniformly with it. If the feed is arrested after a time, and the bar is started revolving, the imaginary worm mounted on it will still be kept in proper mesh with the work, owing to the change gear connections between the cutter-bar and the work spindle, acting through the differential gearing. As previously explained, the office of the differential gearing is to combine in the work the rotation due to the feeding and that due to the rotation of the worm, in such a way that they can take place simultaneously as well as separately; so that it will be seen that if the connections are properly made, the worm may be fed endwise and revolved at the same time, always keeping in perfect step with the work.

Now, the imaginary worm and the fly-tool are both firmly fixed to the cutter-bar, so that the fly-tool must always follow the movements of the imaginary worm. Being set to coincide with the outlines of the worm thread at the start, it must always coincide with those outlines, and since the worm is never out of step with the work, the fly-tool will never be out of step either. It will thus be seen that it will always follow the helical path of the dotted lines in Fig. 5, in moving, for instance, from T_1 to T_3. Revolving in the position T_3, T_4, T_2, etc., the work, as shown in the dotted lines of T_2, will always be in proper relation with the fly-tool, as it is with the imaginary worm.

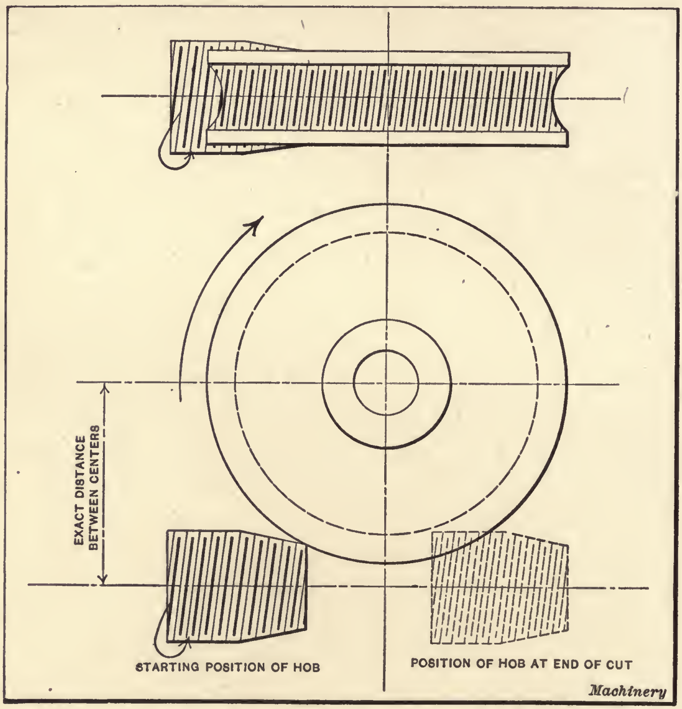


Fig. 6. Diagram Illustrating Manner in which a Tapered Hob is Presented to a Worm-wheel Blank

With this arrangement, if the change gearing connecting the driving mechanism of the cutter-bar and the work were disconnected while the bar were fed through from left to right, the rotary motion given by the connection of the feed of the bar with the work would shape one tooth. If, on the other hand, the gearing connecting the feed of the bar with the rotation of the work were disconnected while the connections between the drive of the bar and the work were in operation, the cutter would partially shape each tooth of the work. By combining the two movements in the differential gearing, the cutter perfectly forms all the teeth.

Tapered Hob for Cutting Worm-wheels. — When cutting worm-wheels by this method, the hob, as indicated in Fig. 6, is tapered. It is placed on the cutter spindle and fed axially

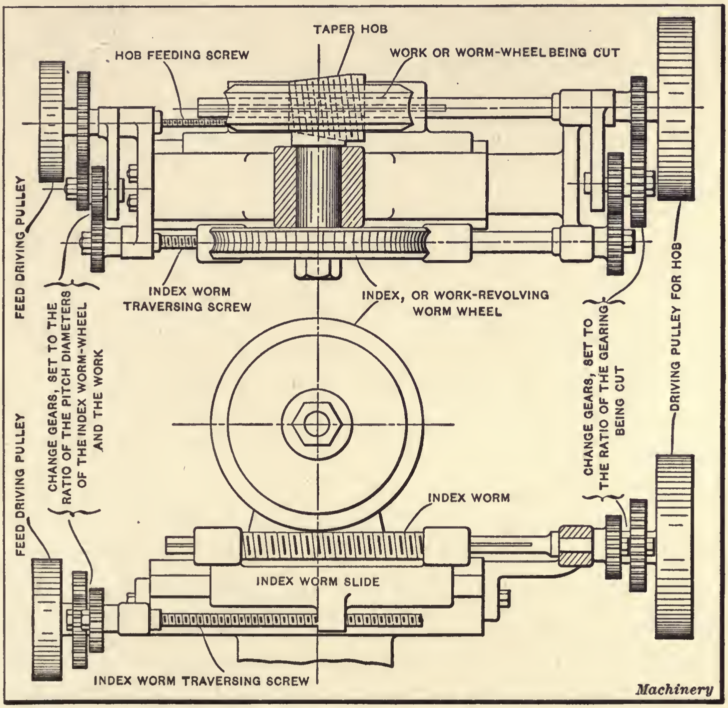


Fig. 7. Diagram of Original Form of Mechanism for Generating Worm-wheels with Taper Hob

past the work in the same way that the fly-tools in the previous case are fed. The combined movements cause the hob to follow spirally in the path of the thread of the imaginary rotating worm. The small end of the hob first commences to work, and as the cutter spindle is fed forward, the cut is taken successively on larger and larger diameters, until finally, when the tool has passed clear through, the full-sized teeth at the rear end of the hob complete the work. The machine used is practically identical with that shown in Fig. 6, Chapter IV, it being adapted to cutting worms by the same process. The differential mechanism used is the same as in the illustration referred to, the axial feed of the cutter spindle being applied to shaft *M*, while the rotative movement of the cutter spindle is connected with shaft *H*, the two being combined in gears *J*, *L* and *N* to rotate the indexing wheel *G*.

The original machine for this purpose, built by Mr. Reinecker, Chemnitz, Germany, employed a different form of combining or differential movement. It is shown diagrammatically in Fig. 7. In this case the tapered hob is connected by change gearing with the worm driving the indexing wheel, as before. The worm, however, is mounted on a slide, allowing it a considerable range of axial movement. This axial movement is controlled by a screw and nut, as shown. This screw is connected by change gearing with the screw by which the taper hob is fed. It will thus be seen that the feeding of the hob rotates the work by shifting the index worm lengthwise, while the rotating of the hob rotates the work through the rotation of the index worm and worm-gear. The two movements are independent of each other, but are combined with the same effect as produced by the "jack-in-the-box" differential gearing previously described. With this arrangement the ratio of table movement and lengthwise worm movement should be proportioned in the ratio of the pitch diameters of the worm-wheel being cut, and the index worm-wheel. The reason for abandoning this construction was doubtless its limited range of movement, which, though sufficient for the hobbing of worm-wheels, was not sufficient (when applied to the universal gear-cutting machine) for cutting spiral pinions of great helix angle.

Ordinary and Taper Hob Method of Hobbing Worm-wheels Compared. — By applying the hob to the work in the method just described, a theoretically correct and accurate worm-gear is produced, which cannot be excelled where high pitch angles or wide wheel faces — wide in relation to the worm — are concerned. By such a method of production the area of contact between the worm and worm-wheel is as large as possible, and provided that a good combination of materials is used, *viz.*, a high grade of phosphor-bronze for the worm-wheel and a good grade of casehardening steel for the worm, this type of gearing shows no appreciable wear under the heaviest loads after having run for a long time. In cutting the worm-wheel in this manner, the reverse curve *AB*, Fig. 8, is actually obtained in the worm-wheel tooth. This curve conforms to the path followed by a tooth of the worm in rotating, and in practice is considered to give a surface contact which is impossible to obtain by hobbing worm-wheels in the ordinary way.

The method ordinarily used in hobbing worm-wheels is to gear up the mechanism driving the hob with that actuating the

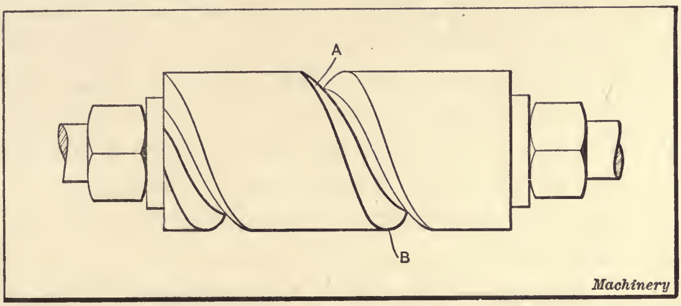


Fig. 8. Diagram Illustrating Reverse Curve that is produced on Worm-wheel Teeth by Hobbing with a Tapered Hob fed Longitudinally and at Right Angles to the Axis of the Worm-wheel Blank

dividing wheel which controls the indexing or rotation of the worm-wheel, and to feed the hob radially into the worm-wheel, both hob and blank being rotated at the same time. This action is continued until the hob has been fed in to the correct depth. Now in analyzing this method of cutting the worm-wheel, it will be seen that by presenting the hob in this manner it does not produce a tooth of a theoretically correct shape, for the simple reason that the hob cuts away certain portions of the tooth that are necessary to give a perfect contact with the worm. This is due to the constant changing of the theoretical helix angle of the hob while being fed in axially.

Instead, therefore, of producing a worm-wheel tooth that has a reverse curve corresponding with the path through which the face of the worm tooth travels in rotating, this method removes a certain amount of the surface of the worm-wheel tooth that should come in contact with the worm, and, in theory, the contact between a worm-wheel (cut in this manner) and the worm is only at the center. The worm-wheel teeth are relieved toward each end and are not in contact with the teeth of the worm, these portions of the worm-wheel teeth being removed by the hob in forming them.

A very high degree of accuracy can be obtained by the taper hob method of hobbing worm-wheels, owing to the fact that the full size of the hob does not come into play until the finishing cut is reached, so that the teeth of the hob tend to preserve their shape indefinitely. Another point that tends to produce accuracy is the fact that the distance between the work arbor and the hob spindle is at all times fixed at exactly the distance between the axis of the worm-wheel and the worm in the finished gearing. This is a refinement of greater importance than is usually realized, and one that is not always looked out for in hobbing operations in which the cutter spindle is fed in toward the work.

Efficiency of Taper-hobbed Worm Gearing. — In order to prove that worm-wheels cut by this method would work out as satisfactorily in practice as theoretical considerations indicated, a number of tests were made by a prominent concern manufacturing pleasure electric cars. In these tests it was found that the efficiency was very high, averaging from 90 to 98 per cent. Continual running appeared to have but very little effect on the efficiency, and the wear was almost negligible; the only effect that wear has on this type of gearing is to increase the backlash between the worm and the worm-wheel teeth. It was also found in these tests that, as far as noise was concerned, the ball bearings used in the design did not run anywhere near as quietly as the worm and worm-wheel, which would indicate that from this point of view the conditions met with in this type of gearing are almost ideal. This type of worm-gearing has also proved highly satisfactory for reduction gearing in connection with electric motors. A particularly good example which illustrates the adaptability of this type of gearing to large reductions was a reduction gear having a ratio of 451.5 to 1 that showed an efficiency of 80 per cent when transmitting 10 horsepower under tests covering a considerable period of time.

Various Methods Compared. — Each of the various methods of cutting worm-wheel teeth described has, however, its field of usefulness. Gashing, as we have seen, is applicable either to cheap, rough-and-ready work on the one hand or, on the other hand, to the cutting of worm-wheels which are not required to transmit a great amount of power, but in which the highest degree of accuracy is required. The process of hobbing previously gashed blanks requires the least degree of specialization in the machinery used, the ordinary milling machine having all the movements and adjustments required. This process is perhaps the one followed in most shops in making worm gearing of small size. The arrangement (such as shown in Figs. 2 and 3) in which the work and the hob are positively geared together so that previous gashing is not required, is quicker than the last-mentioned method, but requires special machines or attachments. The fly-tool method requires a still more elaborate machine, but is the least expensive of all in the matter of cutting tools. A large hob is an exceedingly costly appliance, and raises the cost of production to an alarming degree, particularly when but one worm-wheel has to be cut. The use of a simple fly-cutter, which may be ground accurately to size after hardening so that all inaccuracies are avoided, is thus the cheapest as well as the most accurate means of cutting a large worm-wheel. Where many large wheels of the same size are to be cut, the taper hob method is the most satisfactory one. Hobbing by this method is, of course, more rapid than by the fly-tool process employed on the same machines, though the latter is not a tedious operation by any means, as a solidly supported and powerfully driven tool can be given a heavy feed, taking off chips of considerable thickness.

The Worm. — The methods followed in cutting the other member of this form of gearing, the worm, have already been described in connection with the methods for cutting helical and herringbone gears. A few general remarks may, however, be added. Worm-threads have an included angle between the sides of 29 degrees, as shown by the sectional view, Fig. 9. The width of the worm-thread tool at the end equals the linear pitch *P* of the worm (or circular pitch of the gear) multiplied by 0.31. It is difficult to thread worms having a large lead or "quick pitch" on an ordinary lathe, because the lead-screw must be geared to run several times faster than the spindle, thus imposing excessive strains on the gearing. A common method of overcoming this difficulty is to mount a belt pulley on the lead-screw, beside the change gear and belt it to the countershaft; the spindle is then driven through the change gearing.

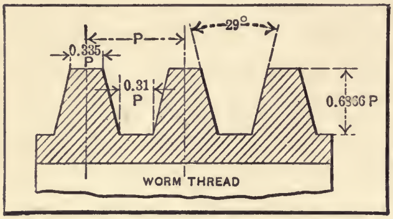


Fig. 9. Dimensions of the Worm Thread

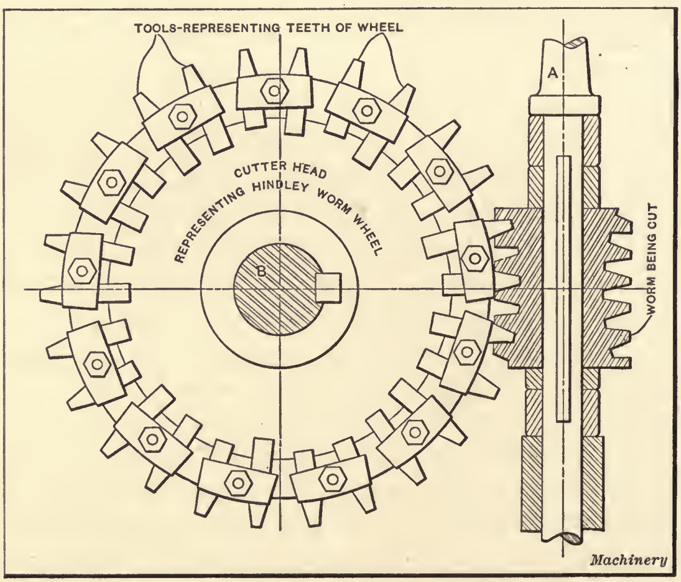


Fig. 10. Method of Cutting Hindley Worm with Rotary Cutter having Teeth Corresponding with Those of the Wheel

It is quite common practice to use a thread milling machine for cutting worms. By means of this machine worm milling becomes quite an easy matter. The machines are simple in operation and good work can be turned out cheaply and satisfactorily. The cutter head is accurately graduated and can be swiveled either way to the correct angle of the thread. The headstock spindle is hollow, allowing work to pass completely through it, and the cross-slide is provided with an automatic stop. The machines can also be stopped automatically, at the end of a thread, thus insuring regularity of length when cutting

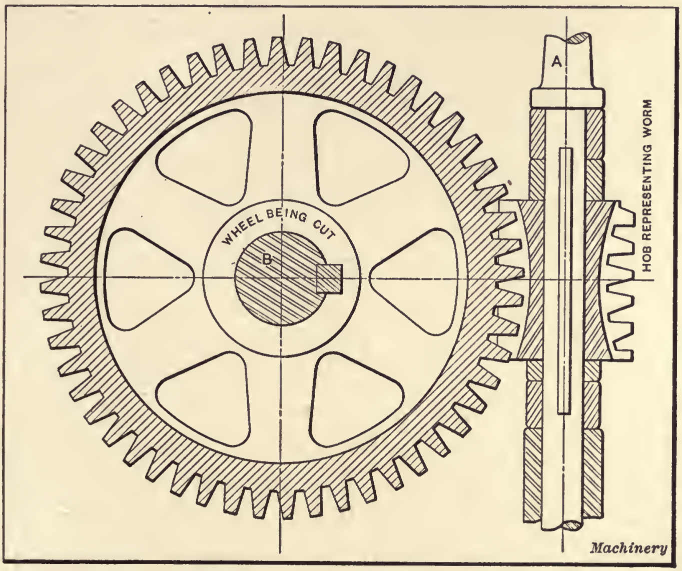


Fig. 11. Cutting the Teeth of the Hindley Worm-wheel with a Hob Corresponding to the Worm

multiple-pitch worms. There is, of course, as is evident in all milled spiral flutes, a tendency to leave the face of the worm-thread cut somewhat concave. In order, therefore, to produce the best results, one maker of worm drives finds it advisable to rough out the worm on the thread milling machine, and leave a grinding allowance of 0.010 inch on each face of the thread. The worm, after hardening, is then ground by a special machine which corrects this concavity of the worm tooth and also removes any distortion which is likely to be caused by hardening.

Manufacture of the Hindley Worm Gearing. — Any positively operated worm-wheel hobbing attachment or machine may be used for cutting Hindley worm gearing. The manufacture begins with the cutting of the worm, which is done as shown in Fig. 10. The blank is mounted on the spindle of the machine ordinarily occupied by the hob, while a large diameter disk provided with cutting tools clamped to its face is mounted in place to represent the worm-wheel. The cutting tools mounted on this disk each represent a tooth of the wheel, being of the same shape and cutting on the same diameter. They are clamped to the face of the disk in such a way that the whole arrangement represents accurately a central section of the worm-wheel, of which (in this particular case) only every other tooth is used. This cutter and the worm to be cut are geared together, and slowly fed toward each other as when hobbing worm-wheels. The teeth, cutting deeper and deeper into the blank, finally form it into the characteristic "hour-glass" shape of the Hindley worm.

In cutting the wheel the process is reversed, as shown in Fig. 11. A hob cut in the same way as the worm in Fig. 10, but with its teeth relieved, is fed into the wheel blank and cuts the teeth in a way exactly identical with the method followed in hobbing worm-wheels, the only difference in the process being the difference in the shape of the hob and in the shape of the teeth produced.

The above paragraphs relate to the principles involved in cutting Hindley worm gearing. The actual operation of hobbing the worm and the gear is a little more complicated in practice, as certain corrections have to be made for interference that require cutting the worm first with a cutter head of the same diameter as the wheel in the way we have shown, and a second time with a head of somewhat larger diameter.

CHAPTER XI

GASHING AND HOBBING A WORM-WHEEL

In the construction of worm gearing the distance from the center of the worm to the center of the worm-wheel may be fixed, or, in some cases, variations, within reasonable limits, may be permitted. When the center distance is fixed, which will be the condition governing the work under consideration, the mechanic may have the opportunity of testing the accuracy of his work by assembling the finished gear in its place, which is, of course, desirable. We shall assume, however, that in this case, such opportunity is not afforded.

The Gashing and Hobbing Process. — The worm itself should first be accurately finished as it can be used advantageously in testing the center distance when hobbing the worm-wheel. We shall assume that this has been done, and that the wheel blank has also been turned, and will consider the method of hobbing the teeth in the latter in a universal milling machine. It is first necessary to gash the blank. This operation, as already indicated in the preceding chapter, consists of cutting teeth, which are approximately the shape of the finished teeth, around the periphery of the blank, by the use, preferably, of an involute gear cutter of a number and pitch corresponding to the number and pitch of the teeth in the wheel. If a gear cutter is not available, a plain milling cutter, the thickness of which should not exceed three-tenths of the circular pitch, may be used. The corners of the teeth of the cutter should be rounded, as otherwise the fillets of the finished teeth will be partly removed. After the gashing operation, the teeth are finished to conform to the shape of the worm by revolving the blank in unison with a cutter known as a hob, sinking the latter into the blank until the teeth are cut to the required depth. As the worm which meshes with and drives the worm-wheel is simply a short screw, it will be apparent that if the axes of the worm-wheel and worm are to be at right angles to each other, the teeth of the wheel must be cut at an angle to its axis in order to mesh with the threads of the worm. The method of setting the work and obtaining this angle will first be considered.

Setting the Work and the Machine. — After the dividing head and tail-stock have been clamped to the table and the cutter has

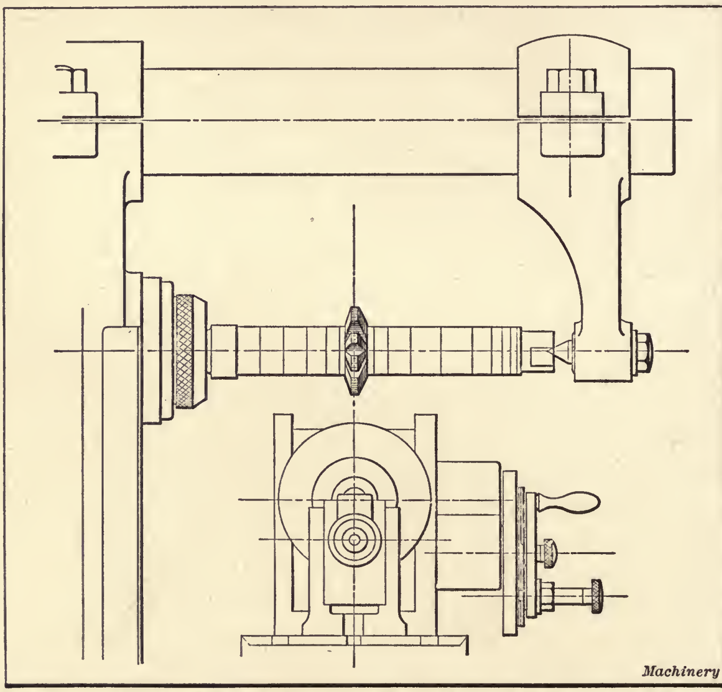


Fig. 1. Setting the Milling Machine for Gashing a Worm-wheel Blank

been fastened on its arbor, the table is adjusted until the point of the center of the dividing head and the center of the cutter lie in the same vertical plane. This may be done by moving the carriage in or out, as the case may require, until the center of the dividing head spindle is directly under the center of the cutter. If a standard gear cutter is used, as in Fig. 1, the center in the head may be set to coincide with a center line on the cutter which is placed there by the makers to facilitate setting the cutter central with the work spindle. If a plain cutter is used (which will be without the center line), a convenient method of setting it is to place an arbor on the head and tail centers; then with the blade of a centering square projecting upward, adjust the carriage until the side of the cutter has a full contact with the central edge of the blade. A second adjustment of the carriage, equal to one-half the thickness of the cutter, will locate the latter central with the dividing head. When the table is set it should be securely clamped to the knee slide.

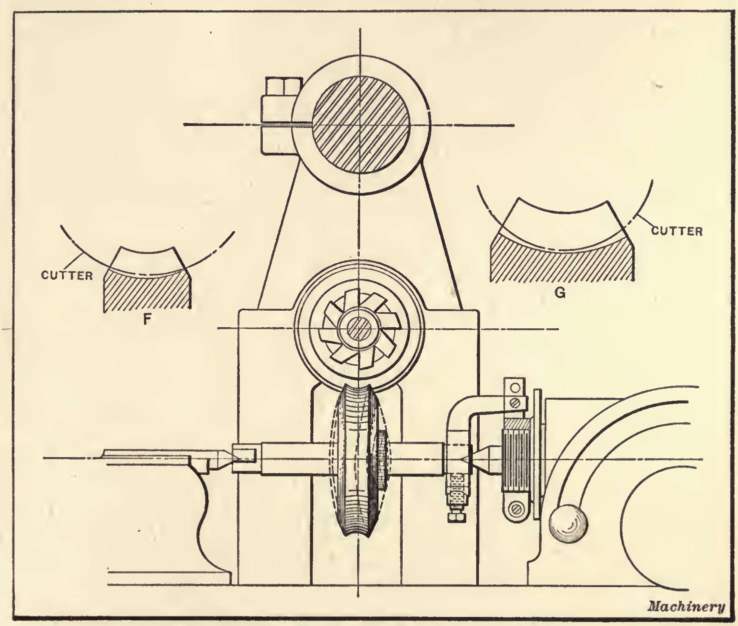


Fig. 2. Gashing the Worm-wheel Blank

The blank to be gashed is now pressed on a true-running arbor which is mounted between the centers of the dividing head and tail-stock as illustrated in Fig. 2, and the driving dog is secured, to prevent any vibration of the work. The table is now moved longitudinally until a point midway between the sides of the blank is directly beneath the center of the cutter arbor. To set the blank, place a square blade against it on first one side and then the other and adjust the table until the distances between the blade and arbor, on each side, are equal. Of course, if the diameter of the arbor were greater than the width of the blank, the measurements would be taken between the latter and the square blade.

Setting the Table of the Machine. — The table should now be set to the proper angle for gashing the teeth. This angle, which should be given on the drawing, may be determined either graphically or by calculation. The first method is illustrated in Fig. 3. Some smooth surface should be selected, having a straight edge as at A. A line B, equal in length to the lead of the worm thread, is drawn at right angles to the edge A, and a distance C laid off equal to the circumference of the pitch circle of the worm.

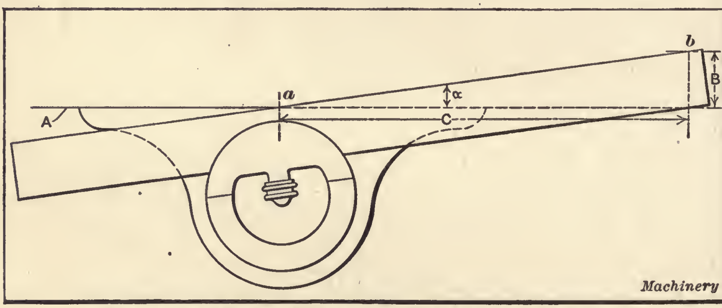


Fig. 3. Method of Obtaining Helix Angle of Worm

If the diameter of the pitch circle is not given on the drawing, it may be found by subtracting twice the addendum of the teeth from the outside diameter of the worm. The addendum equals the linear pitch × 0.3183. The angle α is then accurately measured with a protractor, as shown in the illustration.

The table of the machine is then swiveled to a corresponding angle which can be measured by the graduations provided on all universal milling machines. If the front of the table is represented by the edge A, and the worm has a right-hand thread, the table will be swiveled as indicated by the line ab; if the worm has a left-hand thread the table will be turned in an opposite direction. The angle that the teeth of the worm-wheel make with its axis, or the angle to which the table is to be swiveled, may also be found by dividing the lead of the worm thread by the circumference of the pitch circle; the quotient will equal the tangent of the desired angle. This angle is then easily found by referring to a table of natural tangents.

The Gashing Operation. — The cutter is next sunk into the blank to the proper depth. If the diameter of the cutter is no larger than the diameter of the hob to be used, the depth of the gashes should be just a trifle less than the whole depth of the tooth. This whole depth is found by multiplying the linear pitch by 0.6866, as explained in Chapter VI. When the diameter of the cutter is greater than that of the hob, we have the condition shown at *F* in Fig. 2. The depth to which the gashing cutter should be set is then limited by the depth of the cut at the side of the blank. Should the diameter of the cutter be smaller than that of the hob, the condition shown at *G* is encountered. Here the limiting depth to which the cutter should be set is shown to be on the center line.

Before starting a cut, bring the cutter into contact with the wheel blank and set the dial on the elevating screw at zero. Then sink the cutter to the proper depth, as indicated by the dial. When the cutter is larger than the hob, the depth of the tooth should be laid out on the beveled side of the cutter blank and a gash cut to this line. The depth as indicated on the dial should then be noted and all the gashes cut to a corresponding depth.

The Hobbing Operation. — When the gashing is finished, the table is set at right angles with the spindle of the machine, and the cutter is replaced with a hob as shown in Fig. 4. The outside diameter of the hob and the diameter at the bottom of the teeth are slightly greater than the corresponding dimensions of the worm in order that there may be clearance between the latter and the worm-wheel. Before hobbing the dog is removed from the arbor.

Adjust the table longitudinally until the center of the blank is directly under the center of the arbor *H*. The blank may be set quite accurately by bringing it into contact with the arbor and adjusting the work until the arbor rests centrally in the throat of the blank. With the arbor still in contact with the periphery of the blank, at its throat, set the dial of the elevating screw at zero. Measure the diameter of the arbor with a micrometer, and divide this dimension by 2, obtaining the radius. Measure the diameter of the hob *K*, and also ascertain its radius. Subtract the radius of the arbor from the radius of the hob, lower the knee of the machine an amount equal to this result, and again set the dial at zero. The knee may now be lowered a small amount in order that the blank may clear the hob when the latter is being placed on the arbor.

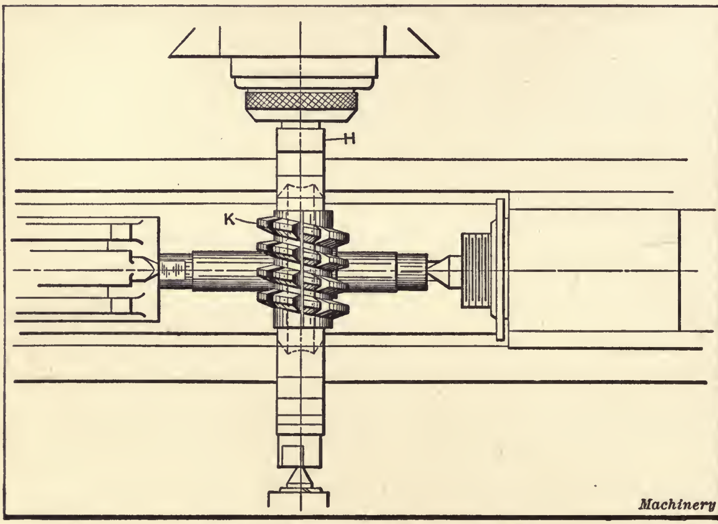


Fig. 4. Hobbing the Worm-wheel

Tighten the hob on its arbor, and then raise the knee until the hob is in mesh with the gashes in the blank. It will be observed that the whole tooth depth has not been reached when the hob bottoms in the gashes. The machine is now set in motion and as the hob revolves the blank rotates with it. The hob is now fed into the blank, by raising the knee, until the dial indicates the correct depth. If the hob is properly made, and the wheel blank accurately sized, the teeth will be cut to the correct depth when the inner diameter of the hob grazes the blank at its throat diameter. The hob and blank should now rotate several times to eliminate any spring and to produce smooth teeth.

After the work has made a few revolutions, to insure well-formed teeth, as mentioned, the hob and wheel are disengaged, and the finished worm is placed in mesh with the latter, as shown in Fig. 5, after the chips have been thoroughly removed from the teeth on which the worm bears. The worm is now turned along the periphery of the wheel until its axis is parallel with the top of the table. It may be set in this position by testing the top surfaces of the threads at either end with a surface gage. Set the pointer of the gage so that it just touches the top of a thread

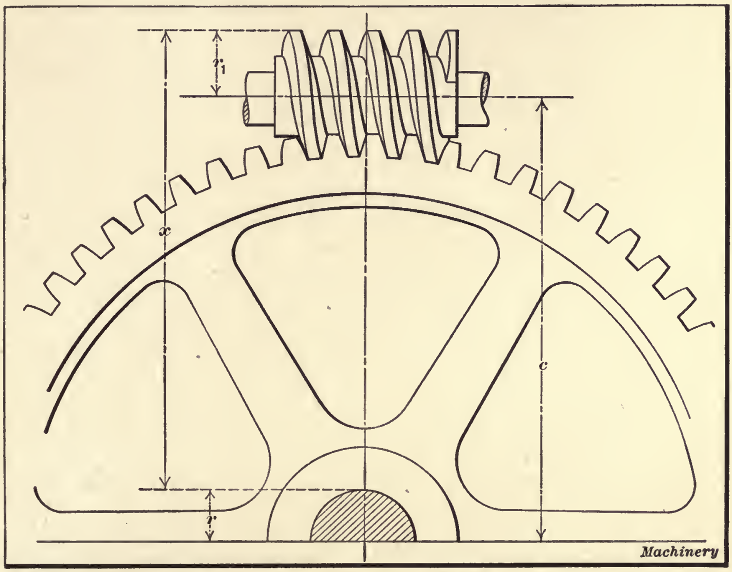


Fig. 5. Determining the Center Distance Between Worm and Wheel

and measure the distance x from the pointer to the arbor. Subtract from this dimension the difference between the radii r and r_1 of the arbor and worm, and the result will be the center distance c. If the worm is accurately made and the worm-wheel blank turned to the exact dimensions, this center distance should be very close to the distance required. If necessary, the hob may be again engaged with the wheel and another light cut taken. When testing the center distance, as explained in the foregoing, it is better to lower the knee sufficiently to make room for the worm beneath the hob, and not disturb the longitudinal setting of the table, as the relation between the wheel and hob will then be maintained, which is desirable in case it is necessary to rehob the wheel to reduce the center distance.

Reducing Flats on Hobbed Worm-wheel Teeth. — The larger the diameter of a hobbed gear — the pitch remaining the same — the more closely the tooth outline approaches the shape of a rack tooth. The flats left on the teeth by the hobbing operation also become perceptibly less as the size of the wheel increases. The flats on the teeth of hobbed worm-wheels with a small number of teeth can be reduced by the use of a hob having a large number of flutes. Where a fly-cutter hob is used, the flats can be further reduced by moving the hob along its axis after the

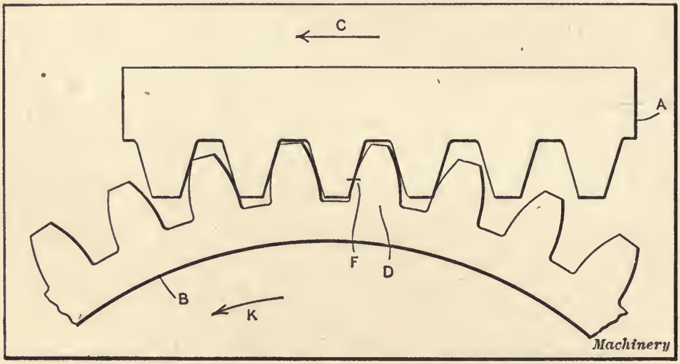


Fig. 6. Hob Working on Worm-wheel in First Position

first cut has been taken and moving the worm-wheel on its arbor a corresponding amount. By making five or six such shifts of the hob, a very smooth worm-wheel is produced. The fly-cutter hob and gear blank must be geared together and the blank cut to depth before shifting the hob on its axis as described.

In Fig. 6 the tooth *D* of gear *B* is in contact with the hob *A* at the point *F*. In this position, a series of flats are produced on the gear as it revolves in the direction indicated by arrow *K*. By moving the hob along its axis a distance equal to a small fraction of the circular pitch in the direction shown by the arrow *C*, the hob *A* is brought into contact with the tooth *D* at the point *H* as indicated in Fig. 7. A new series of flats is produced in this way causing the corners to be sheared off the flats which were produced when the hob was in the position illustrated in Fig. 6. By repeating this process, moving the hob along its axis a number of times, the flats produced in hobbing a worm-wheel in this way can be practically eliminated. The total distance through which the hob is moved is from one to two times the circular pitch. The movement of the hob along its axis can be accomplished automatically by suitable apparatus properly timed and operating in connection with the driving mechanism of the gear and hob. Although it takes slightly longer to hob wheels by this method, the increased accuracy of the work more than compensates for the additional time which is necessary

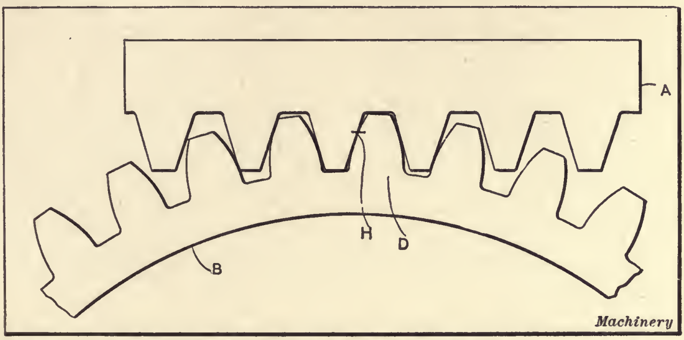


Fig. 7. Conditions after Hob has been Shifted, showing Change in Relative Position of Hob and Wheel

for the operation. This method of gear hobbing is used by the Boston Gear Works when a smooth worm-wheel is required.

Suggested Refinement in the Hobbing of Worm-wheels. — At the left of Fig. 8 is a sectional view showing a hob in the act of putting the last finishing touches on a worm-wheel. The hob is supposed to be a new one and is shown in the condition it is in when first received from the makers. At the right of Fig. 8 is shown the same hob putting the finishing touches on a worm-wheel similar to that in the first case. The hob in this case is represented as having been in use for a considerable time, and having been ground down to the last extremity, ready to be discarded for a new one. A study of this cut will show that if the hob is made in the first place to properly match the worm which is to drive the wheel, it will not, when worn, cut exactly the proper form of tooth in the blank to mesh with that worm. The teeth are cut to the same depth in each case, this being necessary in order to make a proper fit with the worm, which is the same in each case and is set at the same center distance. The grinding away of the worn hob has reduced its diameter by an amount indicated by dimension *b*. Its center is therefore at *P* on the line *AB*, which is offset by a distance represented by dimension *a* from the line *CD* on which the center *O* of the new hob is located. This reduction in diameter as the hob is ground away from time to time, so evidently follows from the construction of the relieved hob, that it scarcely needs to be explained.

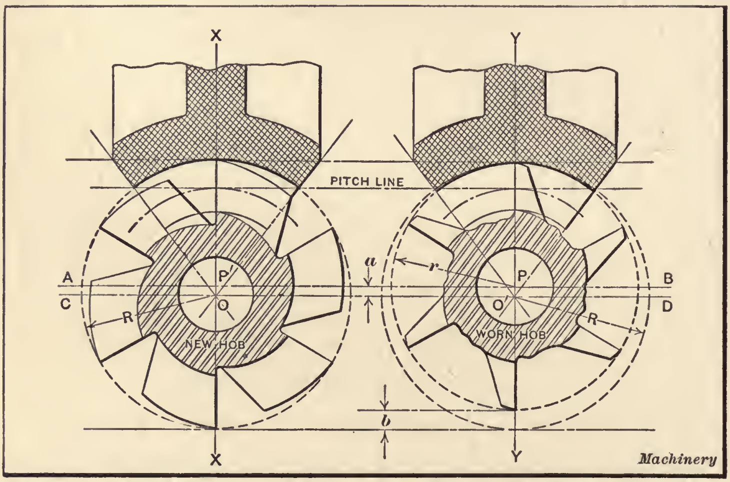


Fig. 8. The Difference in Shape of Teeth Cut by New and Old Hobs

It is said of relieved hobs that they can be ground without changing their shape. This is true so far as the outline of the cutting edge is concerned, but it will be evident, on examining the conditions shown at the right hand of Fig. 8, that whatever the outline of the cutting edges, a new hob of radius *R* will not cut exactly the same shape teeth in the blank as the worn hob with radius *r*. The elements of the tooth surface it generates are struck from a center *P*, removed by dimension *a* from center *O′* which is the location of the axis of the worm with which it meshes.

It is possible, and perhaps practicable, to overcome this slight error; that is, to so design and use the hob that it will cut as correct teeth when worn as when new. In Fig. 9 dotted line *AA* represents the outlines of a new hob in the act of finishing the worm-wheel shown. Were a hob, ground as shown at the right of Fig. 8, to be substituted on the arbor for this new hob, without altering the adjustment of the machine except to move the hob endwise and bring it in contact with the teeth of the wheel on one side, this hob would be represented in Fig. 9 by the full line *BB*. It is evident that the left-hand cutting edges of this hob coincide (to the depth they extend into the wheel) with those of the new hob represented by outline *AA*. They will, therefore, so far as they extend, cut identically similar and correct tooth curves with the new hob.

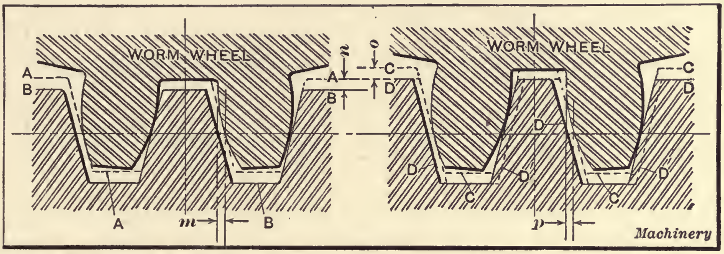


Fig. 9. Cutting Action of Ordinary Hob at Fixed Center Distance when New and when Worn

Fig. 10. Action of Proposed Hob when New and when Old Graphically Shown

Teeth cut with this worn hob would, however, evidently have two faults. The space would be too narrow at the pitch line by a distance measured by dimension *m*, and they would not be cut deep enough in the blank by a distance measured by dimension *n*. The problem is to so alter the design and application of the hob, that, even when worn, we can cut the teeth deep enough and the space wide enough.

Fig. 10 shows these conditions fulfilled. Dotted line *CC* shows the outline of the proposed hob when new. The only difference between the proposed hob and the regular one, whose outlines are shown by the dotted line *AA* in Fig. 9, is that the teeth have been lengthened by an amount equal to dimension *o*. The hob is fed in as was the case with the new hob in Fig. 9 until the distance between its center line and that of the blank is the same as that between the center line of the worm and the wheel in the finished machine. The increase in radius, then, by an amount *o*, makes the hob cut a clearance deeper than is necessary by that amount. In a spur gear this would doubtless be a bad thing, since it would make the tooth slenderer and therefore weaker. A worm-gear, however, if designed to be sufficiently durable for continuous use, is almost certain to be several times stronger than necessary, so that the slight weakening involved in the change is not of great importance. When the hob is worn to the shape shown by the full outline *DD*, the hob is evidently of the same diameter as the new one in Fig. 9, represented by dotted outline *AA*. The tooth space, however, as before explained, will be too narrow by the amount *m* in Fig. 9 or *p* in Fig. 10. To widen it out sufficiently, it is, therefore, necessary, after the hob has been fed in to the proper depth, to still continue the cutting action, feeding the hob endwise, however, until it has been displaced to the position indicated by outlines *D′D′*. The resulting tooth is evidently identical with that given by the new hob *AA* in Fig. 9.

It will be understood that when the hob in Fig. 10 is new, it will not have to be shifted end-wise at all, since it will cut a tooth space of the proper width as soon as fed to depth. It will, however, cut a space deeper than necessary by an amount *o*. The worn hob, on the other hand, has to be shifted longitudinally by an amount *p* and cuts to exactly the required depth. These represent the two extreme conditions. When the hob is half worn, the excess clearance will be equal to half of *o*, and the longitudinal displacement necessary will be equal to half of *p*.

While the change in the design of the hob could be made easily enough, there is doubtless some difficulty in making the required change in the hobbing of the blank. Taking it for granted that the hob has been made to suit the worm which is to be used, and that it, therefore, has the same pitch diameter and thickness of tooth at the pitch line, the method of procedure will invariably require that the hob be fed in to the worm-wheel blank until the distance from the center of the hob to that of the wheel is the same as the distance from the center of the worm to that of the wheel in the finished machine. This will be true whether the hob is new or worn, and whatever may be the kind of machine on which the hobbing is done.

The method by which the hob is displaced longitudinally will depend on the machine used for the operation. There will be no possible way of doing it if the wheel is being finished while running loosely on centers, as is common practice when the blank has first been gashed. It is required that the hob and blank be positively geared together. If a positively driven hobbing attachment in the milling machine is being used, the matter is simple. If the hob is being driven by the spindle of the machine, throw in the cross feed in either direction until the required longitudinal displacement of the wheel with relation to the hob has taken place. The question as to when this has taken place may be decided either by measuring the thickness of the tooth, as in cutting spur gears, or by trying the wheel from time to time with its worm, the two parts being mounted in place in the machine they are to go in, or held the proper distance apart by other means.

For regular hobbing machines, as at present made, the matter is more difficult. The required longitudinal displacement of the hob may be obtained, in effect, by a rotary displacement of the hob which may be accomplished by slipping (a tooth at a time), the teeth of the change gears connecting the hob and the blank. If a hobbing machine were to be built especially for use in the way which is here suggested, differential gearing could be introduced in the train between the hob and the wheel, to which a power feed could be given to effect the rotary displacement when the hob has been fed to depth, or a power feed might be applied to feed the spindle and its attached hob endwise to effect the same result.

It is not certain that the error which exists in the use of relieved hobs is of enough importance to warrant taking any trouble to remedy it. It is always well, however, to know and understand such errors as may exist in any process of this sort, no matter if they are of no great practical importance.

CHAPTER XII

HOBS FOR WORM-GEARS

If a worm and gear of standard proportions are brought into mesh, we have at the bottom of both the thread of the worm and teeth of the gear a clearance equal to one-tenth of the thickness of the thread or tooth at the pitch line. The clearance at the root of the gear tooth is obtained by enlarging the hob over the diameter of the worm, by an amount equal to two clearances, while the clearance of the tooth in the thread bottom is

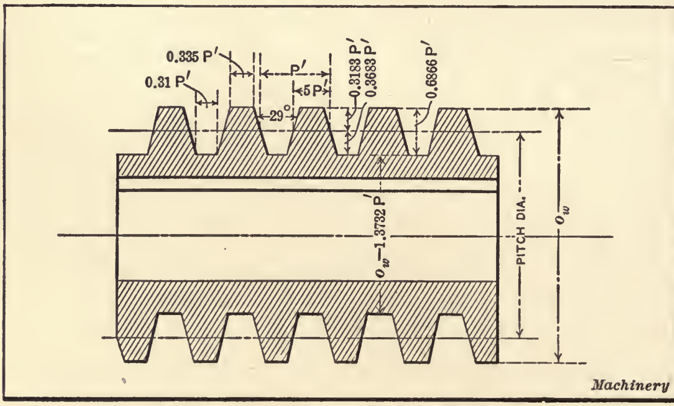


Fig. 1. Dimensions of the Worm

taken care of by the proper sizing of the gear blank.

Dimensions of Hob. — While it may be customary practice to make the hob an exact duplicate of the worm except in the one item of outside diameter, a hob proportioned as suggested in Fig. 3 is recommended as one that will give much more satisfactory results, and be found to be well worth any additional trouble in construction required beyond that for the style ordinarily used. The peculiar feature of this hob is that it is an exact opposite of the worm with respect to the proportions of the thread shape; the depth below the pitch line in one case being equal to the height above the pitch line in the other. The object of this is to have a hob that will form the complete outline of the tooth and make it absolutely certain that the standard proportions of tooth and clearance are obtained. Thus, should the diameter of the blank be large, the hob will trim off the top of the gear teeth to the proper length, when the proper center distance is maintained.

There is another point that is generally overlooked, and that is the necessity for having the corners of the thread rounded over, and for providing a liberal fillet at the root of the thread. The radii of the rounded corner and the fillet may be as large as the clearance will allow, which would be one-twentieth of the circular pitch of the thread.

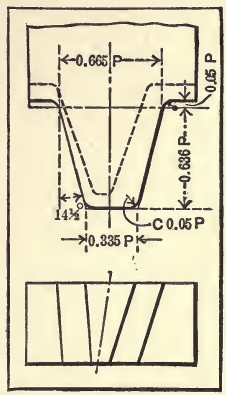


Fig. 2. Dimensions of Tool for Threading the Hob

The effect that this fillet and rounded thread has on the shape of the tooth is due to the fact that it increases the quality of the gear and the strength of each individual tooth. The rounded corner on the thread points does away with any tendency to scratch the surface of the tooth in the cutting action, and leaves a much larger fillet at the root, greatly increasing the strength. The fillet at the bottom of the thread rounds off the top of the tooth in the worm-gear, removing any burrs, and leaving a nicely finished product. This fillet also removes the dangerous tendency of the hob to develop cracks in the hardening process — a common source of trouble even where care is taken. Fig. 1 shows the proportions of the worm in comparison with the hob in Fig. 3.

Thread Tool for Hob. — In forming the hob much can be gained by making a special form tool of correct proportion that will leave no chance for error; the only dimension needing care, then, is the diameter. Such a tool is shown in Fig. 2. The figure is dimensioned by formulas, so that a tool for any pitch can be easily proportioned from it. This tool may be made by using a gear caliper without resorting to the protractor, or the protractor may be used in laying out the angle. This tool may be made without side clearance, providing that the sides incline in the same direction and at the same angle that the thread takes, but, under ordinary circumstances, where only one hob is to be made, little is gained by having no side clearance. The clearance may be made from 5 to 10 degrees from the angle of the thread. Grinding a tool like this of course changes its form, so it must not be used indefinitely in making large numbers of similar hobs.

Number of Flutes in Hobs. — The number of flutes that should be provided in the hob is a point on which very little is said, various authorities differing widely. Where the hob is to

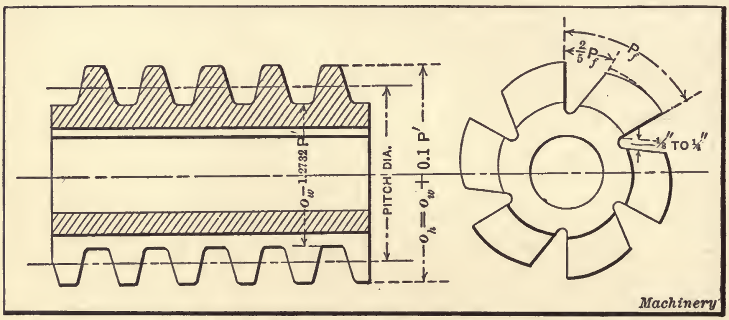


Fig. 3. Dimensions of the Worm-wheel Hob **Fig. 4. Dimensions for Fluting the Hob**

be used in an automatic hobbing machine in which the hob and blank are positively geared together, the number of flutes may be a comparatively small number as compared with a hob that is to be used in connection with ordinary processes of hobbing worm gears. In the process in which the previously gashed worm-gear blank is swung loosely on centers and revolved by the hob as the latter rotates, the hob should have a larger number of flutes.

A rule that checks up well with present practice is as follows: *To find the number of flutes in a hob, multiply the diameter of the hob by three, and divide by twice the circular pitch.*

The above rule gives suitable results on hobs for general purposes.

Some authorities on worm-gearing state that the number of flutes in a hob should in no case be an exact multiple of the number of threads. Their reason for this rule is that the hob so gashed will produce a much smoother tooth and one nearer correct in shape, because no tooth in the hob passes the same tooth in the gear twice in succession, so that any imperfections in the shape of the individual hob teeth are counteracted by one another. Another authority is strong in his advice not to have the circumferential distance from flute to flute equal to or equally divisable by the circular pitch, for the same reason as stated regarding the former rule. From these statements it is seen that to obtain a rule that would be at once simple and yet take all conditions into consideration, would be a difficult proposition. It seems, however, that only the first of these two rules is a logical one. Owing to the fact that hobs have teeth only, instead of full surfaces matching the worm, the curved outlines of the wheel teeth are merely approximated by a series of tangents. If the number of flutes in the hob is a multiple of the number of threads, the hob teeth will "track" after each other, giving wheel teeth only roughly approximated by a comparatively small number of long tangents. This subject is treated in detail in the latter part of this chapter.

Character of Flutes. — The cutter used in gashing the hob should be about $\frac{1}{8}$ inch thick at the periphery for hobs of ordinary pitch, while for those of coarser pitch a cutter $\frac{1}{4}$ inch thick would be much better. The width of the gash at the periphery of the hob should be about two-fifths the pitch of the flutes. The cutter should be sunk into the blank so that it reaches from $\frac{3}{16}$ to $\frac{1}{4}$ inch below the root of the thread. Fig. 4 shows an end view of a hob gashed according to these rules.

Where a hob is to be used to any great extent, and is subject to much wear, it would be advisable to increase the diameter over the dimensions given from 0.010 to 0.030 inch, according to its diameter and pitch, to allow for decrease in diameter due to the relief, and caused by grinding back the cutting face in sharpening.

Hobs are generally fluted parallel with the axis, but it is obvious that they should be gashed on a spiral at right angles with the thread helix in order that the cutting face may be presented with theoretical correctness; but the trouble encountered in relieving the teeth on the ordinary backing-off attachment is the cause of the common mode of fluting. When the pitch or lead is coarse in comparison with the pitch diameter of the hob, so that the angle is correspondingly steep, it may be best to flute on the normal helix, and if the hob cannot be machine relieved, it may be backed off by hand.

The amount of relief depends much on the use for which the hob is intended. A hand hob for hobbing a gear in position may

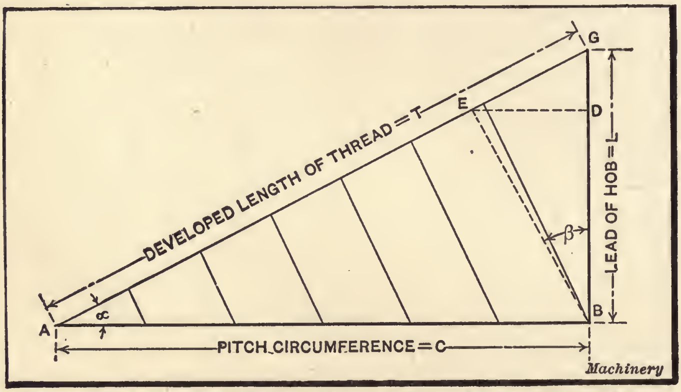


Fig. 5. Diagram for the Derivation of Formula for Spirally-fluted Hobs

be made with little or no relief, while hobs used on hobbing machines may have much more relief than those used on the milling machine.

Spiral-fluted Hob Angles. — It is, of course, desirable that hobs should be fluted at right angles to the direction of the thread. Sometimes, however, it is necessary to modify this requirement to a slight degree, because the hobs cannot be relieved unless the number of teeth in one revolution, along the thread helix, is such that the relieving attachment can be properly geared to suit it. In the following it is proposed to show how an angle of flute can be selected that will make the flute come approximately at right angles to the thread, and at the same time the angle is so selected as to meet the requirements of the relieving attachment.

Let C = pitch circumference;
T = developed length of thread in one turn;
N = number of teeth in one turn along thread helix;
F = number of flutes;
α = angle of thread helix.

Then (see Fig. 5):

$C \div F$ = length of each small division on pitch circumference.

$(C \div F) \times \cos\alpha$ = length of division on developed thread. $C \div \cos\alpha = T$.

$$\text{Hence } \frac{T}{(C \div F)\cos\alpha} = N = \frac{F}{\cos^2\alpha}$$

Now, if $\alpha = 30$ degrees, $N = 1\frac{1}{3}F$;
$\alpha = 45$ degrees, $N = 2F$;
$\alpha = 60$ degrees, $N = 4F$.

In most cases, however, such simple relations are not obtained. Suppose for example that $F = 7$, and $\alpha = 35$ degrees. Then $N = 10.432$, and no gears could be selected that would relieve this hob. By a very slight change in the spiral angle of the flute, however, we can change N to 10 or $10\frac{1}{2}$; in either case we can find suitable gears for the relieving attachment.

The rule for finding the modified spiral lead of the flute is: *Multiply the lead of the hob by F, and divide the product by the difference between the desired value of N and F.*

Hence, the lead of flute required to make $N = 10$ is:

Lead of hob $\times (7 \div 3)$.

To make $N = 10\frac{1}{2}$, we have:

Lead of flute = lead of hob $\times (7 \div 3.5)$.

From this the angle of the flute can easily be found.

That the rule given is correct will be understood from the following consideration. Change the angle of the flute helix β so that AG contains the required number of parts N desired. Then EG contains $N - F$ parts; but $\cot\beta = BD \div ED$, and by the law of similar triangles:

$$BD = \frac{F}{N} \times BG, \text{ and } ED = \frac{N-F}{N}C.$$

The lead of the spiral of the flute, however, is $C \times \cot\beta$.

Hence, the required lead of spiral of the flute:

$$C \times \cot \beta = \frac{F}{N - F} L.$$

This simple formula makes it possible always to flute hobs so that they can be conveniently relieved, and at the same time have the flutes at approximately right angles to the thread.

Graphical Method. — The angle of the flutes, determined so as to avoid difficulties in relieving, may be found graphically as

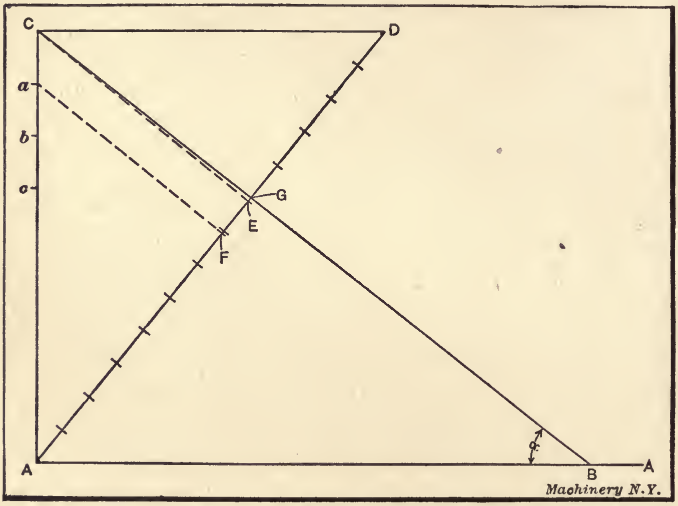


Fig. 6. Graphical Method for finding Gashing Angle and Number of Flutes for which Backing-off Attachment should be Set for Spirally-fluted Hobs

follows: First, lay off a base line *AA*, Fig. 6, of any convenient length. Then erect the perpendicular *AC* making it equal to the developed length of the pitch circumference of the hob. From *C* draw line *CD* parallel to the base line *AA* and of a length equal to the lead of the hob. Now draw diagonal *AD* which represents the thread. Divide *AC* into as may equal parts as there are flutes in the hob, as *a*, *b* and *c*. From *C* and *a* draw lines through and at right angles to the diagonal *AD*, as *CE* and *aF*. Then length *EF* equals the pitch of the flutes on the thread when the gashing is at right angles to the thread. To proceed, divide *AD* into a certain number of equal parts, the length of these parts to be as near to the length EF as possible. Step off these divisions on AD, and through the division nearest to E, as at G, draw a line from C to the base line intersecting the base line at B. This line CB represents the gash, line AB the lead of the gash, and the number of divisions in the line AD equals the number of flutes to one revolution of the hob, for which we must gear the machine.

To get the exact length of AB, divide the number of divisions in AG by the number of divisions in GD and multiply the result by the length of the line CD or the lead of the hob. The angle α which is the angle for gashing can be found by scaling the diagram. For example, let the hob be 2 inches pitch diameter, lead 5 inches, and number of flutes 8.

We first draw base line AA, and the line AC 6.28 inches long which is the pitch circumference. Now draw CD 5 inches long, and then draw line AD. We now divide AC into eight equal parts and draw lines from C and a through and at right angles to AD, intersecting AD at E and F. Setting the dividers to length EF we step off line AD and find that this length EF will go into AD a little over thirteen times; so we divide this line AD into thirteen equal parts. It is now necessary to gear the machine for thirteen flutes to one revolution of the hob.

The division nearest to E is G, so by drawing a line from C through G we intersect the base line at B. In the line GD there are five divisions, and in the line AG there are eight divisions. The lead of the hob is five inches, so that the length of the lead for the gash or AB is $\frac{8}{5} \times 5 = 8$ inches. By measuring on the diagram by a scale we find the gashing angle is $38\frac{1}{4}$ degrees. Therefore, we will gear the machine used in backing-off the hob for 13 flutes to one revolution, and we will gear the milling machine to cut a lead of 8 inches, at a gashing angle of $38\frac{1}{4}$ degrees.

Lengths of Worms and Hobs. — The derivation of a formula for quickly finding the approximate length of a hob, or of a worm to run with a worm-wheel having hobbed teeth, depends primarily upon an assumed relation between the worm and worm-wheel and also upon the pitch and size of the wheel. The relation between the length of worm and the dimensions of the worm-wheel differs with the conditions under which the gears are to run or the standards of the manufacturer.

In Fig. 7 the hob (or worm) is shown extending from A to B, which produces a hob having the maximum generating action. This length also provides a safe allowance for any end adjustment which may be necessary for the worm. A hob to cut a worm-wheel without interference should be as long as the worm to be used, and neither should be less than the length of the chord between points F and G, which are on the wheel pitch circle.

Let AB or length of hob $= f$;

BC or throat circle radius $= r$;

DE or whole depth of tooth $= d$;

Number of teeth in worm-wheel $= N$.

$$CE = BC - DE = r - d.$$

Solving the right-angle triangle enclosed by the lines BC, CE and BE $\left(\text{or } r,\ r - d \text{ and } \frac{f}{2}\right)$, we have:

$$(r-d)^2 + \left(\frac{f}{2}\right)^2 = r^2;$$

$$r^2 - 2\,rd + d^2 + \frac{f^2}{4} = r^2.$$

$$d^2 + \frac{f^2}{4} = 2\,rd; \qquad \frac{f^2}{4} = 2\,rd - d^2.$$

$$f^2 = 4\,d\,(2\,r - d); \qquad f = 2\sqrt{d\,(2\,r - d)}$$

In order to further simplify the formula, we will assume the pitch to be 1-inch circular pitch.

DE or d (whole depth of tooth) would then equal 0.6866 inch, and BC or r (throat circle radius) would be equal to $\dfrac{N+2}{2 \times 3.1416}$.

Substituting we get:

$$f = 2\sqrt{0.6866\left(\frac{N+2}{3.1416} - 0.6866\right)}$$

This can be simplified as follows:

$$f = 2\sqrt{\frac{0.6866\,(N+2)}{3.1416} - (0.6866)^2}$$

$$f = 2\sqrt{0.21855\,N - 0.03432}$$

Squaring both sides of the equation we get:

$$f^2 = 0.8742\,N - 0.13728.$$

As the value for f need be only approximate, we can write the equation as follows:

$$f^2 = \frac{7N}{8} - \frac{11}{80};$$

$$f^2 - \frac{7N}{8} + \frac{11}{80} = 0;$$

$$80f^2 - 70N + 11 = 0.$$

Now, if we solve for f for different numbers of teeth, the length of hob or worm for 1-inch circular pitch will be obtained.

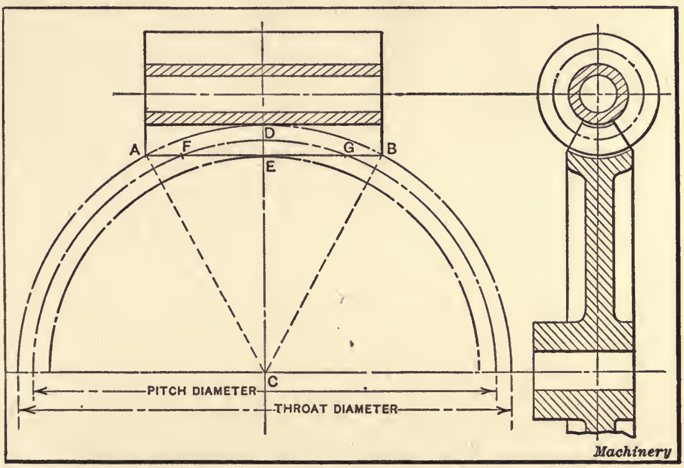


Fig. 7. Diagram of Worm and Worm-wheel for Determining Length of Worm and Hob

For other pitches it will be necessary to multiply by the circular pitch to obtain the correct length.

The accompanying table gives the values of f for 1 inch circular pitch. To illustrate the use of this table, suppose we desire to find the length of a worm to suit a $\frac{3}{4}$-inch circular pitch worm-gear, having 39 teeth. Find the value for f in the table opposite 39 teeth. This value is 5.83 and multiplied by the pitch, $\frac{3}{4}$ inch, gives 4.37, or about $4\frac{3}{8}$ inches, which is the length of the worm or hob.

Table of Constants for Determining the Lengths of Worms or Hobs

Factor *f* equals length of worm when circular pitch is 1 inch. To find length for any other pitch, multiply factor *f* corresponding to given number of teeth in worm-wheel by the required pitch.

No. of Teeth in Worm-wheel	Factor *f* for 1-inch Circular Pitch	No. of Teeth in Worm-wheel	Factor *f* for 1-inch Circular Pitch	No. of Teeth in Worm-wheel	Factor *f* for 1-inch Circular Pitch	No. of Teeth in Worm-wheel	Factor *f* for 1-inch Circular Pitch	No. of Teeth in Worm-wheel	Factor *f* for 1-inch Circular Pitch
10	2.93	44	6.18	78	8.25	112	9.92	146	11.28
11	3.08	45	6.25	79	8.30	113	9.96	147	11.32
12	3.22	46	6.32	80	8.35	114	10.00	148	11.36
13	3.35	47	6.39	81	8.40	115	10.04	149	11.40
14	3.48	48	6.46	82	8.45	116	10.08	150	11.44
15	3.60	49	6.53	83	8.50	117	10.12	151	11.48
16	3.72	50	6.60	84	8.55	118	10.16	152	11.52
17	3.84	51	6.67	85	8.60	119	10.20	153	11.56
18	3.95	52	6.74	86	8.65	120	10.24	154	11.60
19	4.06	53	6.80	87	8.70	121	10.28	155	11.64
20	4.17	54	6.86	88	8.75	122	10.32	156	11.68
21	4.27	55	6.92	89	8.80	123	10.36	157	11.72
22	4.37	56	6.98	90	8.85	124	10.40	158	11.755
23	4.47	57	7.04	91	8.90	125	10.44	159	11.79
24	4.57	58	7.10	92	8.95	126	10.48	160	11.825
25	4.66	59	7.16	93	9.00	127	10.52	161	11.86
26	4.75	60	7.22	94	9.05	128	10.56	162	11.895
27	4.84	61	7.28	95	9.10	129	10.60	163	11.93
28	4.93	62	7.34	96	9.15	130	10.64	164	11.965
29	5.02	63	7.40	97	9.20	131	10.68	165	12.00
30	5.11	64	7.46	98	9.25	132	10.72	166	12.035
31	5.20	65	7.52	99	9.30	133	10.76	167	12.07
32	5.28	66	7.58	100	9.35	134	10.80	168	12.105
33	5.36	67	7.64	101	9.40	135	10.84	169	12.14
34	5.44	68	7.70	102	9.45	136	10.88	170	12.175
35	5.52	69	7.76	103	9.50	137	10.92	171	12.21
36	5.60	70	7.82	104	9.55	138	10.96	172	12.245
37	5.68	71	7.88	105	9.60	139	11.00	173	12.28
38	5.76	72	7.94	106	9.65	140	11.04	174	12.315
39	5.83	73	8.00	107	9.70	141	11.08	175	12.35
40	5.90	74	8.05	108	9.75	142	11.12	176	12.385
41	5.97	75	8.10	109	9.80	143	11.16	177	12.42
42	6.04	76	8.15	110	9.84	144	11.20	178	12.455
43	6.11	77	8.20	111	9.88	145	11.24	179	12.49

Number of Flutes in Hobs. — The question of how many gashes to cut in a worm hob, particularly if the hob is multiple threaded, has always been a puzzling one for most mechanics. Many believe that the only requirement is that the number of gashes must have no common factor with the number of threads in the worm. That is to say, if the worm is quadruple threaded, the number of gashes should be 9 or 7 rather than 8. If the worm is sextuple threaded, the number of gashes should be 7 or 11 rather than 8, 9 or 10. This is one requirement, but there seem to be other factors that enter into the decision as well. These were brought to the attention of Mr. R. E. Flanders, who carefully investigated this subject a few years ago, by Mr. N. B. Chace, superintendent of the Cincinnati Shaper Co., who was endeavoring to obtain a hob that would cut smooth, regular teeth for the worm-wheel of the spindle drive in a machine he was building.

The worm-wheel of this drive had 35 teeth. The worm had 7 threads and a lead of 5 inches. The number of flutes or gashes in the hob was 9. These gashes were milled spirally so that they were at right angles to the thread. The hob was made by a well-known firm which makes a specialty of such work; it was proved by subsequent tests to be accurately and finely made, and altogether a very creditable piece of work. Do what he could, however, Mr. Chace was unable to hob worm-wheels that would be satisfactory. When tried in place in the machine and run with the worm, each one appeared to have five low spots, almost as if the pitch line were a pentagon instead of a circle. The wheels were taken out and the thickness of the teeth in the center of the throat at the pitch line measured as accurately as possible.

The results of one of these tests are shown in Fig. 8, where it will be seen that there is a regular recurrence of thin teeth in each fifth of the circumference of the wheel with less marked series of fine intermediate thin teeth. The diagram in the center of the figure shows graphically (by the exaggerated radial distance from the center) the variation in the measurements obtained. The first thought would naturally be that the hob had warped out of true in hardening, in which case the ratio between the worm and the wheel of 5 to 1 would give the error indicated; but careful measurements failed to detect any error of this kind, either in the periphery of the hob or on the sides of the cutting edges.

The Imperfect Generating Action of the Hob. — To find what was really the trouble with the hob (or rather, with the work of

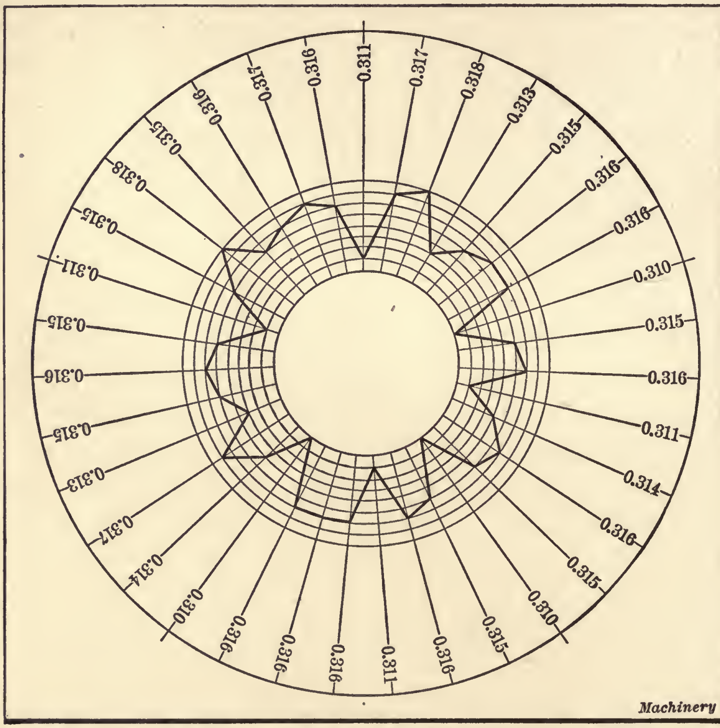


Fig. 8. Thick and Thin Teeth Produced by Incomplete Generating Action on the Part of the Hob

the hob, for the hob was found to be all right), it will be necessary to study its action in cutting a worm-wheel. The diagram in Fig. 9 will serve to illustrate some of the important points connected with this action. In the upper part of the diagram at the right is shown an end view of a single-threaded hob having six gashes. To the left of this is shown the pitch cylinder of the hob with a helix traced upon it, representing the center

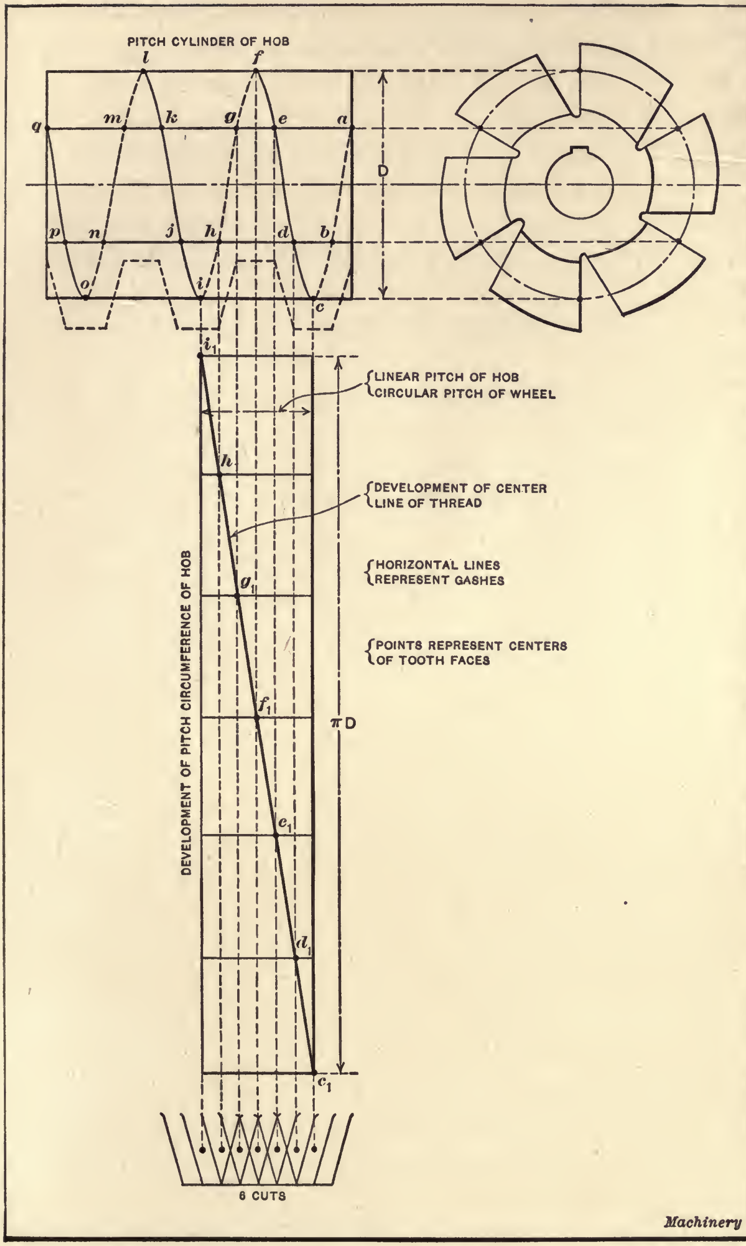


Fig. 9. Finding the Number of Cuts per Linear Pitch

of the thread. Lines parallel with the axis of the work are drawn on this pitch cylinder, representing the intersection of the faces of the teeth with the cylinder. The intersections of the helix with these lines at *a*, *b*, *c*, *d*, etc., represent the positions on the pitch cylinder of the center of each of the teeth of the hob.

Below this representation of the pitch cylinder is shown a development of its circumference through an axial length equal to the linear pitch of the worm, represented in this case by the distance *ci*. On this development, the tooth helix between *c* and *i* becomes a straight line, as shown, and the center of the tooth faces *c*, *d*, *e*, *f*, *g*, *h* and *i* are developed, as before, by the intersection of this tooth line with equally spaced horizontal lines representing the six gashes in the circumference. Below this development of the circumference of the hob is shown a series of outlines of the cutting edges of the hob, each one of which has its center directly below the corresponding center c_1d_1, etc., in the development. These outlines evidently represent the successive positions of the teeth of the hob as they pass the plane of the throat of the worm-wheel in hobbing its teeth. There are seven of these positions, but as one of them belongs to the next section of the hob, from *i* to *o*, the diagram shows six positions of the hob teeth in the linear pitch of the hob.

This means, of course, that the hob does not accurately generate a tooth of the wheel, since it acts on it only in the six successive positions shown, instead of continuously throughout the whole distance of the circular pitch. In order to get smooth accurate teeth, the number of cuts in the linear pitch must be made as many as possible; the more there are, the more nearly perfect would the generating action be; the less there are, the rougher will be the tooth. Now, as will be shown later, there is but *one* cut per linear pitch in the example mentioned in the paragraphs headed "Number of Flutes in Hobs." Under these circumstances the teeth of the worm, instead of being smoothly generated to a curve, are only slabbed out by a series of flat cuts, as indicated in Fig. 10.

The reason for the five thin teeth in the circumference is now evident. At every fifth of a revolution those teeth of the hob which formed the outline near the pitch line of the gear gave it a shape similar to that shown at the right of the engraving. In the thick teeth the conditions shown at the left are found, where the outline of the tooth at the pitch line is formed by cuts so placed as to make corners at this point instead of flats as at the right. This means that the teeth measured on the pitch line are thick at one point and thin at the other, giving high spots, as found by running the wheel with the worm, and as indicated also by the measurement shown in Fig. 8. The reason for the intermediate thin spots between the even fifths is not clear from

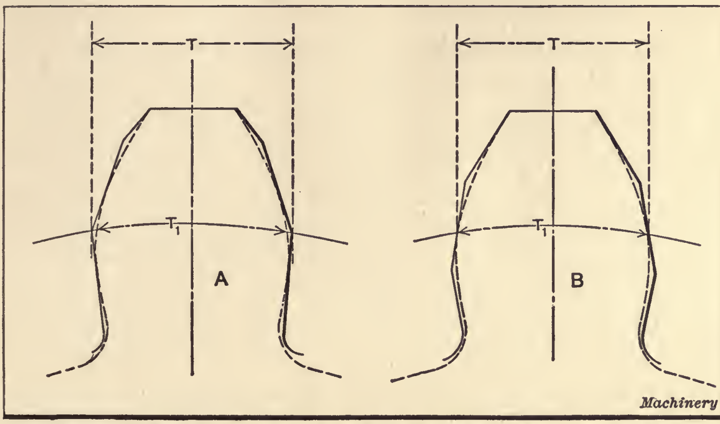


Fig. 10. Example of Thick and Thin Teeth due to Incomplete Generating Action

the preceding explanation, but they are doubtless due to the particular arrangement of the flats on the tooth outline which happens in this particular wheel.

Diagrams for Finding the Number of Cuts per Linear Pitch. — It is evidently a simple matter to draw diagrams for any case showing the development of one linear pitch on the pitch surface of the hob, as in Fig. 9, and find out from that diagram how many cuts the hob gives in that distance. In Fig. 11 eight such diagrams are shown, for eight different cases. The first case is a single-threaded hob having five gashes. This diagram, which is similar to the one in Fig. 9, shows that there are five cuts to the linear pitch. In the second diagram a hob of the same diameter and the same linear pitch having also five gashes, but quintuple instead of single threaded, gives but one cut to the linear pitch. This is evidently a very bad condition and one to be avoided, if possible, and it is evidently brought about from the fact that the number of gashes is the same as the number of threads.

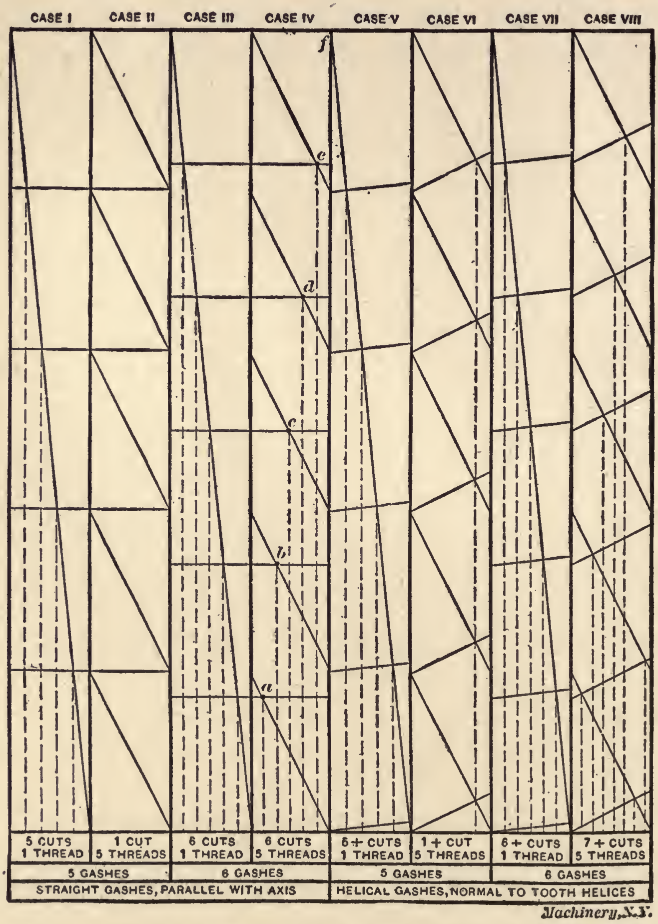


Fig. 11. Diagram for Finding the Number of Cuts per Linear Pitch

In the third and fourth cases the number of gashes has been increased to six, with a single thread in one case and a quintuple thread in the other. In each case there are six cuts to the linear pitch. The fifth and sixth cases are the same as the first and second, except that the lines representing the gashes have been drawn at right angles to the lines representing the tooth helices as would be necessary for hobs which are gashed helically in a direction normal to the tooth helices. These cases will be seen to correspond to Nos. 1 and 2 except that the number of cuts has been increased in proportion to the cosine of the gashing angle, so that we have 5 + cuts for Case V, and 1 + cuts for Case VI. In Cases VII and VIII are shown the same conditions as in Cases III and IV, except that the hob is gashed helically. In this case, also, the number of cuts is increased in inverse proportion to the cosine of the gashing angle, giving 6 + and 7 + cuts, respectively, for the two cases, the hobs having six gashes each.

In Fig. 12 are shown four more cases, considerably more complicated than those in Fig. 11. Here are four hobs, all of the same linear pitch and pitch diameter, and all octuple threaded, with threads of the same lead and helix angle, the only difference in the four being in the number of gashes and the method of cutting them. In Cases IX and XI there are eleven gashes, and in Cases X and XII there are twelve. Cases IX and X are gashed parallel with the axis. This, of course, would be utterly impracticable in any hob having threading angles as great as those shown here, so the example is not a practical one, being used only for the sake of illustrating a principle. Cases XI and XII which are gashed helically and normally at right angles to the threads, represent what would be the practical construction of these hobs. Projecting the intersections of the thread lines with the gash lines, down to the bottom of each diagram, we get for Case IX, eleven cuts to a linear pitch; for Case X, three cuts to a linear pitch; for Case XI, 38 + cuts; and for Case XII 11 + cuts.

The Effect of the Number of Teeth in the Wheel. — There is still another factor entering into this problem — the number of teeth in the wheel. This is the factor which gave so much trouble in the case mentioned. Take, for instance, Case IV in Fig. 11. Suppose that the quintuple-threaded, six-gashed hob, represented by that diagram, were cutting a 25-tooth wheel, it would not give the six cuts indicated by the diagram. The reason for this will appear by comparing Case IV with Case III.

In Case III, where the hob is single threaded, all of the cuts represented by the points of the intersections of the thread and gash lines, are along the same thread.

In Case IV, however, each of the five thread lines in the diagram has but one intersection. That means that if the number

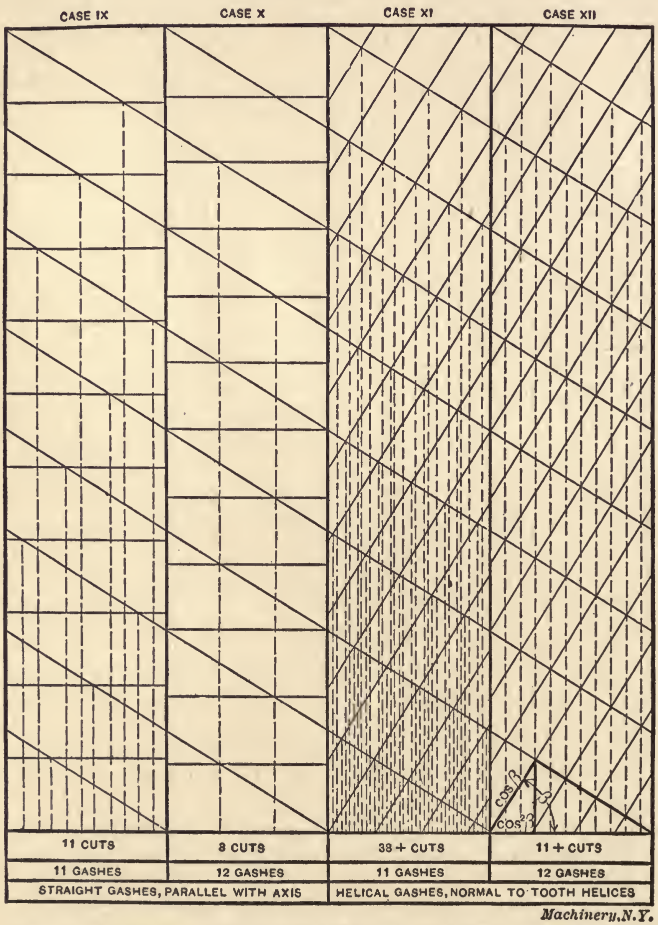


Fig. 12. Diagram for Finding the Number of Cuts per Linear Pitch

of teeth in the gear, as in the supposed example, is a multiple of the number of threads in the worm or hob, each of those threads will come back into the same tooth spaces in the wheel at each revolution of the latter, so that for each tooth space there is but one cutting position of the hob tooth — that represented, for instance, by point *a* for one of the tooth spaces, point *b* for the next, *c* for the next, and so on. If, on the other hand, there were 26 teeth in the wheel, the first time it went around, point *a* would cut in a certain tooth space; the second time around point *b* would come in the same space, and the third time around point *c* would follow, so that each tooth space would get the benefit of each one of the six cuts, the same as in the single-thread worm for Case III. It is thus seen that, besides the other points mentioned, the number of teeth in the wheel has an effect on the number of cuts of the worm per linear pitch. In the practical case previously mentioned there was a 35-tooth worm-wheel and a 7-threaded worm, giving the worst conditions possible.

A General Formula for Determining the Number of Cuts. — From the preceding description it will be seen that there are three points to be taken into consideration in determining the number of cuts per linear pitch (and the consequent generating efficiency of the worm) from the number of gashes in the hob. These factors are: First, the relation of the number of threads of the hob to the number of gashes. Second, the angle of the gashing. Third, the relation of the number of threads of the hob to the number of teeth in the wheel to be cut. It might be considered that there is a fourth factor, that of the absolute number of teeth in the wheel, since the trouble that comes from a small number of cuts per linear pitch is exaggerated in the case of a wheel having very few teeth. This is not a matter of calculation, however, and would not enter into the calculations anyway, since for any given case for which a hob is being designed, the number of teeth in the wheel is determined approximately at least.

Now, instead of drawing diagrams such as shown in Figs. 9, 11 and 12, it would be better if a simple mathematical expression could be obtained which would give the number of cuts per linear pitch directly. This can easily be done. The effect of the number of gashes with relation to the number of threads is as follows: *The number of cuts per inch varies inversely with the greatest common divisor of the number of threads and the number of gashes in the hob.* The influence of the number of teeth in the wheel is a similar one and may be expressed as follows: *The number of cuts per linear pitch varies inversely with the greatest common divisor of the number of threads in the worm and the number of teeth in the wheel.* The effect of the angle of the gashing may be expressed as follows: *The number of cuts per linear pitch varies inversely with the square of the cosine of the gashing angle, measured from a line parallel with the axis of the hob.* These statements are combined in the following formula:

$$X = \frac{G}{D \times D' \times \cos^2 \beta}$$

in which

G = number of gashes;
β = angle of gashing with axis;
D = G. C. D. of number of threads and number of gashes in hob;
D' = G. C. D. of number of threads in hob and number of teeth in wheel;
X = number of cuts per linear pitch.
(G. C. D. = "greatest common divisor.")

It is easier to see the relationships expressed above, from the foregoing diagrams and description, than it is to explain them. These relationships, although quite simple, are rather elusive. Perhaps, however, the effect of the angle will be understood from the figuring of the triangle at the base of the diagram for Case XII. Note that the formula is true only for the usual cases in which the gashing is either helical and normal to the threading, or straight and parallel to the axis. In the latter case, $\cos^2 \beta = 1$, since $\beta = 0$ deg., and the effect of the angle disappears.

Applying the formula to the practical example already mentioned, we have the following values:

$G = 9$;
$\beta = 20$ deg. (assumed, as the angle was not given);
$D = 1$ = G. C. D. of 9 (number of gashes) and 7 (number of threads);
$D' = 7$ = G. C. D. of 7 (number of threads) and 35 (number of teeth in wheel).

Solving for the number of cuts per inch, we have:

$$X = \frac{9}{1 \times 7 \times 0.9397^2} = \frac{9}{6.181} = 1.45.$$

If the number of teeth in the wheel had been 36 instead of 35, the number of cuts would have been:

$$X = \frac{9}{1 \times 1 \times 0.9397^2} = \frac{9}{0.883} = 10.19,$$

which, it will be seen, would immeasurably improve conditions, giving a fine, smooth outline for this number of teeth in the wheel. In the actual wheel, as cut by the hob, the slab-sided effect shown in Fig. 10 was very noticeable, there being about three cuts to each face of the tooth.

Hobbing Methods which give a Complete Generating Action. — It should be noted that while this faulty generating is liable to occur with hobbing by the usual method of sinking the cutter in to depth in a blank, the same difficulty does not occur in the fly-tool process or in a machine using a taper hob fed axially past the work, as described in a preceding chapter. In the case of these machines, working with either taper hobs or fly-cutters, the number of cuts per pitch is, at the least calculation, the number of revolutions per linear pitch of advance of the cutter spindle; it thus runs up into the thousands, where the diagrams shown in Figs. 9, 11 and 12 give only from 1 to 38.

This treatment of the hob question takes care of all the factors which enter into a determination of the number of gashes to use in a hob, so far as this effects the accuracy of the generating action. Expressed briefly, the conclusions are:

Avoid having a common factor between the number of threads and the number of gashes in the hob.

Avoid having a common factor between the number of threads in the hob, and the number of teeth in the wheel.

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